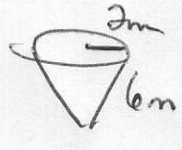
MATH 151 | **QUIZ 4** | NAME Answers
Mrs. Bonny Tighe | 25 points
3.9,3.10 | SECTION __________ Fri 3/10/06

1. Water is being pumped into a conical tank. The tank has height of 6 m and the radius of the base is 2 m. How fast is the volume of the water increasing when the height of the water is rising at 2 m per minute, and the water is 3 m deep? $V = \frac{1}{3}\pi r^2 h$



$r = \frac{1}{3}h$

Given: $dh/dt = 2\text{ m/min}$
Find: dV/dt
when: $h = 3\text{ m}$

$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h = \frac{\pi}{27}h^3$

$dV/dt = \frac{\pi}{9}h^2\, dh/dt$

$dV/dt = \frac{\pi}{9}(3)^2(2) = 2\pi\ \text{m}^3/\text{min}$

2. A spherical snowball is melting so that its volume is decreasing at a rate of $20\ \text{cm}^3/\text{sec}$. Find the rate at which the radius is decreasing when the radius is 5 cm.
$V = \frac{4}{3}\pi r^3$

Given: $dV/dt = 20\ \text{cm}^3/\text{sec}$
Find: dr/dt
when $r = 5\text{ cm}$

$dV/dt = 4\pi r^2\, dr/dt$

$20 = 4\pi(5)^2\, dr/dt$

$\boxed{-\frac{1}{5\pi}\ \text{cm/sec} = dr/dt}$

3. Use linear approximation to estimate $\sqrt{25.5}$ (use 25)

$L(x) = f(a) + f'(a)(x-a)$ or
$y - y_1 = m(x - x_1)$

$f(x) = \sqrt{x} = 5$
$f'(x) = \frac{1}{2\sqrt{x}} \approx \frac{1}{10}$

$y - 5 = \frac{1}{10}(x - 25) = y - 5 = \frac{1}{10}(25.5 - 25)$

$y = 5 + \frac{1}{10}\left(\frac{1}{2}\right) =$

$5\frac{1}{20}$

4. Use linear approximation to estimate $\tan 61°$

$61° = \left(\pi/3 + \frac{\pi}{180}\right)$

use $60° = \pi/3$

$f(x) = \tan x \qquad \tan 60° = \sqrt{3}$

$f'(x) = \sec^2 x \qquad \sec^2 60° = 4$

$y - \sqrt{3} = 4(x - \pi/3)$

$y = \sqrt{3} + 4\left(\pi/3 + \pi/180 - \pi/3\right)$

$y = \sqrt{3} + 4\left(\frac{\pi}{180}\right)$

$f'(x) = 3\sin^2 x(\cos x)$

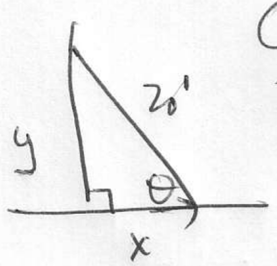
5. Find the linearization, L(x), for f(x) at a when f(x)= $\sin^3 x$ and $a = \pi/6$.

$L(x) = (\sin \pi/6)^3 + 3(\sin \pi/6)^2 \cos \pi/6 (x - \pi/6)$

$= \left(\frac{1}{2}\right)^3 + 3(1/2)^2(\sqrt{3}/2)(x - \pi/6)$

$\boxed{L(x) = \frac{1}{8} + \frac{3\sqrt{3}}{8}(x - \pi/6)}$

6. A 20-foot ladder rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 4 ft/sec, how fast is the angle between the bottom of the ladder and the ground decreasing when the top of the ladder is 8 feet from the ground?



Given: $dx/dt = 4$ ft/sec

Find: $d\theta/dt$

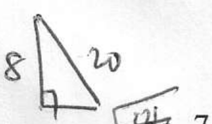
when: $y = 8'$

$\cos\theta = \frac{x}{20}$

$-\sin\theta \, d\theta/dt = \frac{1}{20} dx/dt$

$-8/20 \left(\frac{d\theta}{dt}\right) = -1/20(4)$

$-\frac{1}{2}$ rad/sec $= d\theta/dt = -1/2$ rad/sec



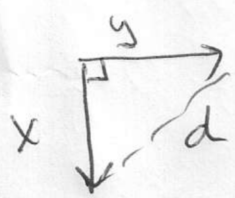
7. Two hikers start walking from the same point. One is walking south at 4 mph and the other is walking east at 3 mph. At what rate is the distance between the two hikers increasing one hour later?

Given $dx/dt = 4$ mph

& $dy/dt = 3$ mph

Find dd/dt

when $x = 4$m & $y = 3$m



$d^2 = x^2 + y^2$

$2d\, dd/dt = 2x\, dx/dt + 2y\, dy/dt$

$2(5)\, dd/dt = 2(4)(4) + 2(3)(3)$

$10\, dd/dt = 32 + 18$

$dd/dt = 5$ mph