

MATH 151
Mrs. Bonny Tighe

QUIZ 3A
3.3-3.5
25 points

NAME Answer
SECTION _____ Fri 2/24/06

1. Find dy/dx or $f'(x)$.

a) $f(x) = 2x^3 - 4x + \frac{1}{x^3} + 4 = 2x^3 - 4x + x^{-3} + 4$

$$f'(x) = (6x^2 - 4 - 3x^{-4}) + 0 = \boxed{6x^2 - 4 - \frac{3}{x^4}}$$

b) $f(x) = 3x\sqrt{x} - \frac{1}{x^2\sqrt{x}} = 3x^{3/2} - x^{-5/2}$

$$f'(x) = 3\left(\frac{3}{2}\right)x^{1/2} - \left(-\frac{5}{2}\right)x^{-7/2} = \boxed{\frac{9}{4}\sqrt{x} + \frac{5}{2x^{7/2}}}$$

c) $y = \frac{3\sec x - \sin x}{\sin x - x^3}$

$$\frac{dy}{dx} = \frac{(3\sec x - x^3)(3\sec x \tan x - \cos x) - (3\sec x \sin x)(\cos x - 3x^2)}{(\sin x - x^3)^2}$$

f) $f(x) = (3x^4 - \csc x)\left(\frac{3}{x^2} + 2x^3\right) = (3x^4 - \csc x)(3x^{-2} + 2x^3)$

$$f'(x) = (3x^4 - \csc x)(-6x^{-3} + 6x^2) + \left(\frac{3}{x^2} + 2x^3\right)(12x^3 + \csc x \cot x)$$

$$= (3x^4 - \csc x)\left(-\frac{6}{x^3} + 6x^2\right) + \left(\frac{3}{x^2} + 2x^3\right)(12x^3 + \csc x \cot x)$$

$$f'(x) = \cos x - \csc^2 x$$

2. If $f(x) = \sin x + \cot x$, find the following:

a)

$$f\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}$$

b)

$$f\left(-\frac{\pi}{6}\right) = -\frac{1}{2} - \frac{\sqrt{3}}{2}$$

c)

$$f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} - 2$$

d)

$$f'\left(\frac{\pi}{2}\right) = -1$$

a) $\sin \frac{\pi}{3} + \cot \frac{\pi}{3}$

$$\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}$$

$$\sqrt{3}/2 + \sqrt{3}/3$$

b) $\sin(-\frac{\pi}{6}) + \cot(-\frac{\pi}{6})$

$$-\frac{1}{2} + (-\sqrt{3})$$

$$-\frac{1}{2} - \sqrt{3}$$

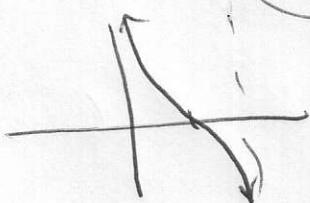
c) $\cos(\frac{\pi}{4}) - \csc^2 \frac{\pi}{4}$

$$\frac{\sqrt{2}}{2} - (\sqrt{2})^2$$

d) $\cos \frac{\pi}{2} - \csc^2 \frac{\pi}{2} = 0 - (1)^2 = -1$

3. Find the limit.

a) $\lim_{x \rightarrow 0^+} (\cot x) = +\infty$



b) $\lim_{x \rightarrow 0} \frac{\sin 4x}{x} = 4$

$$4 \lim_{x \rightarrow 0} \frac{\sin 4x}{4x} = 1 \text{ (u)}$$

c) $\lim_{\alpha \rightarrow 0} \frac{1 - \cos \alpha}{\sin 3\alpha} = 0$

$$\lim_{\alpha \rightarrow 0} \frac{\frac{1 - \cos \alpha}{\alpha}}{\frac{\sin 3\alpha}{3\alpha}} = \frac{\infty}{\infty} \left(\frac{1}{3} \right) = 0$$

4. Find an equation of the tangent to the curve $y = x(\sqrt{x} - 1)$ at the point (4,4).

$$m = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{3}{2}x^{1/2} - 1 \quad \text{at } x=4$$

$$\frac{3}{2}\sqrt{4} - 1 = 3 - 1 = 2$$

$$y = x^{1/2} - x$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 2(x - 4)$$

$$y = 2x - 4$$

5. If $f(3) = 2$, $g(3) = -1$, $f'(3) = 1$ and $g'(3) = 3$, find the following:

a) $(f+g)'(3) = 4$

b) $(fg)'(3) = 5$

c) $\left(\frac{g-f}{g}\right)'(3) = 7$

a) $\frac{f'(3) + g'(3)}{1+3}$

b) $f'(3)g(3) + g(3)f'(3)$

$$2(-1) + (-1)(1)$$

$$6 - 1$$

c) $\frac{g(3)(g-f)'(3) - (g-f)(3)(g'(3))}{(g(3))^2} = \frac{-1(3-1) - (-1-2)(3)}{(-1)^2}$

$$\frac{-1(2) - (-3)(3)}{1} = -2 + 9 = 7$$