

1. Use the **definition** of a derivative to find the slope of the tangent line and use it to find an equation of the tangent line to the curve at the given point.

$f(x) = x - 3x^2 + 4$  at  $(1, 2)$

$$f'(x) = \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1} \frac{(x - 3x^2 + 4) - 2}{x - 1}$$

$$\lim_{x \rightarrow 1} \frac{-3x^2 + x + 2}{x - 1} = \lim_{x \rightarrow 1} \frac{(-x + 1)(3x + 2)}{x - 1}$$

$$\lim_{x \rightarrow 1} -(3x + 2) = \textcircled{-5}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -5(x - 1)$$

$$y = -5x + 7$$

2. The cost of wheat, in dollars per ton,  $t$  days after the harvest, is given by  $W=p(t)$ . What is the meaning of the derivative  $p'(t)$ ? What are its units?

The rate that the cost of wheat is changing in \$ per ton per day

3. The position function of a particle after time  $t$  seconds is given by  $s(t) = -t^2 - 8t + 10$ .

a) Find the average velocity of the particle from  $t = 0$  to  $t = 1$  seconds.

$$\frac{s(1) - s(0)}{1 - 0} = \frac{1 - 10}{1} = \textcircled{-9 \text{ m/sec}}$$

$$s(0) = 10$$

$$s(1) = 1$$

b) Find the instantaneous velocity of the ball at 1 sec., using the definition of derivative to find the slope.

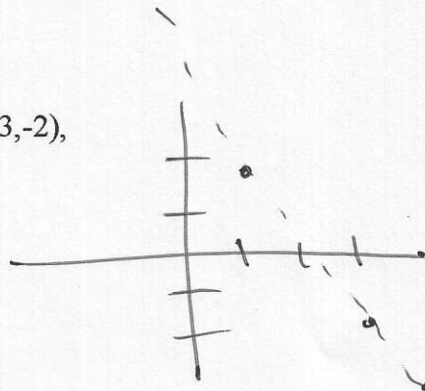
$$\lim_{t \rightarrow 1} \frac{f(t) - f(1)}{t - 1} = \lim_{t \rightarrow 1} \frac{-t^2 - 8t + 10 - 1}{t - 1} = \lim_{t \rightarrow 1} \frac{-(t^2 + 8t - 9)}{t - 1} = \frac{-(t+9)(t-1)}{t-1}$$

$$\lim_{t \rightarrow 1} -(t+9) = \textcircled{-10 \text{ m/sec}}$$

c) Explain why the answers vary by so much.

4. If the tangent line to  $y = f(x)$  at  $(1, 2)$  passes through the point  $(3, -2)$ , find  $f(1) = 2$  and  $f'(1) = -2$ .

$$m = \frac{-2 - 2}{3 - 1} = \frac{-4}{2} = -2$$



5. Find the derivative of the given functions using the **definition of derivative**. State the domain of the function and the domain of the derivative.

a)  $h(x) = \frac{2}{3-x}$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{2}{3-(x+h)} - \frac{2}{3-x}}{h}$$

$$\lim_{h \rightarrow 0} \frac{2(3-x) - 2(3-x-h)}{(3-(x+h))(3-x)h}$$

$$\lim_{h \rightarrow 0} \frac{6 - 2x - 6 + 2x + 2h}{(3-x-h)(3-x)h}$$

$$\lim_{h \rightarrow 0} \frac{2h}{(3-x-h)(3-x)h} = \frac{2}{(3-x)(3-x)}$$

$$h'(x) = \frac{2}{(3-x)^2}$$

b)  $f(x) = 2\sqrt{x+4}$

$$\lim_{h \rightarrow 0} \left( \frac{2\sqrt{x+h+4} - 2\sqrt{x+4}}{h} \right) \left( \frac{\sqrt{x+h+4} + \sqrt{x+4}}{\sqrt{x+h+4} + \sqrt{x+4}} \right)$$

$$\lim_{h \rightarrow 0} \frac{2(x+h+4 - (x+4))}{h(\sqrt{x+h+4} + \sqrt{x+4})}$$

$$\lim_{h \rightarrow 0} \frac{2h}{h(\sqrt{x+h+4} + \sqrt{x+4})}$$

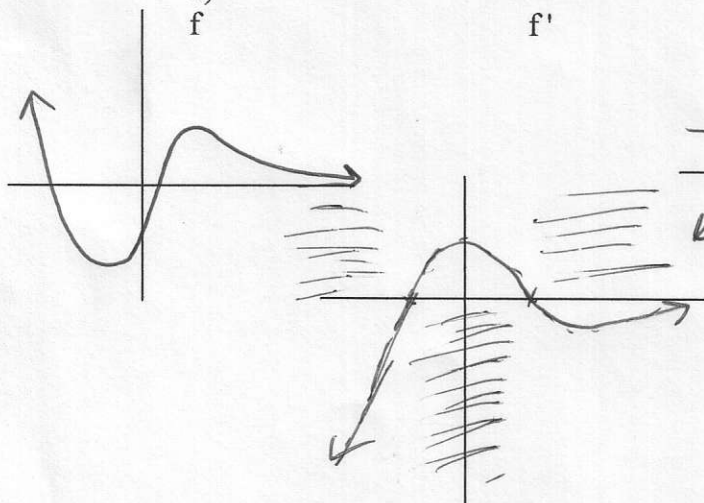
$$\lim_{h \rightarrow 0} \frac{2}{\sqrt{x+h+4} + \sqrt{x+4}} =$$

$$\frac{2}{\sqrt{x+4} + \sqrt{x+4}} = \frac{2}{2\sqrt{x+4}}$$

$$f'(x) = \frac{1}{\sqrt{x+4}}$$

6. Sketch  $f'$  beside each function  $f$ .

a)



b)

