MATH 151
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QUIZ 1A

2.1-2.4
25 points

NAME Answers

SECTION __________ Fri 2/10/06

1. The displacement in meters of a particle moving in s straight line is given by $s(t) = t^2 - 4t + 5$, where t is measured in seconds. a) Find the average velocity of the particle over the time interval $1 \leq t \leq 2$ and then on the interval $2 \leq t \leq 3$. b) Estimate the instantaneous velocity of the particle when t=2 sec.

$s(1) = 1-4+5 = 2$
$s(2) = 4-8+5 = 1$
$s(3) = 9-12+5 = 2$

a) $\frac{s(2)-s(1)}{2-1} = \frac{1-2}{1} = -1$ m/sec

$\frac{s(3)-s(2)}{3-2} = \frac{2-1}{1} = 1$ m/sec

b) 0 m/sec

2. Find the following limits:

a) $\lim_{x\to 0} (\sin^2 x - \cos 3x) =$ -1

$0^2 - 1$

b) $\lim_{x\to 3} \frac{x-3}{9-x^2} =$ $-1/6$

$\lim_{x\to 3} \frac{x-3}{(3+x)(3-x)} = \lim_{x\to 3} \frac{-1}{3+x} =$

c) $\lim_{x\to 1} \frac{1-x}{\sqrt{x}-1} =$ -2

$\frac{(1+\sqrt{x})\cancel{(1-\sqrt{x})}}{\cancel{\sqrt{x}-1}}$ -1

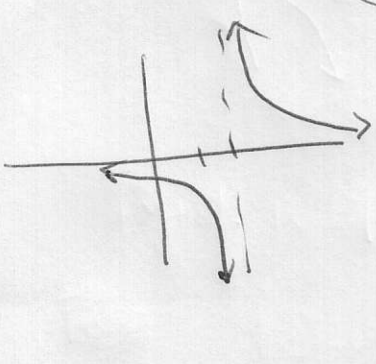
$\lim_{x\to 1} -1(1+\sqrt{x}) =$

d) $\lim_{h\to 0} \frac{(3+h)^{-1} - 3^{-1}}{h} =$ $-1/9$

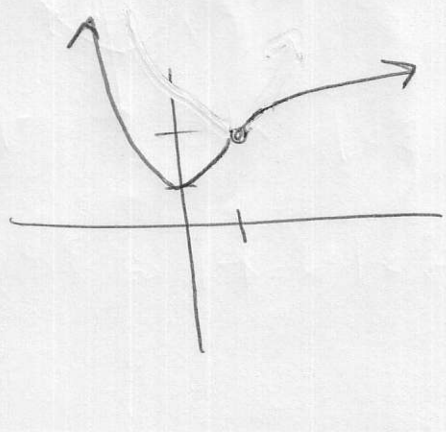
$\lim_{h\to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h} = \lim_{h\to 0} \frac{3-(3+h)}{3(3+h)h} = \frac{-\cancel{h}}{3(3+h)(\cancel{h})}$

$\lim_{h\to 0} \frac{-1}{3(3+h)} = -1/9$

e) $\lim_{p\to 2} \frac{2}{p-2} =$ DNE



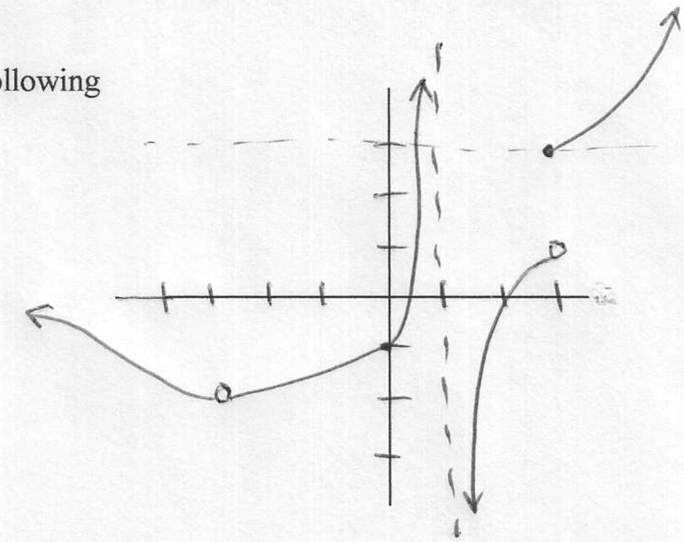
f) $\lim_{x\to 1^+} f(x) =$ 2 if $f(x) = \begin{cases} x^2+1 & \text{if } x<1 \\ \sqrt{x+3} & \text{if } x \geq 1 \end{cases}$



3. If $\lim_{x\to 2} f(x) = -2$ and $\lim_{x\to 2} g(x) = 3$, find $\lim_{x\to 2} \left(\sqrt{f^2 + 1/g}\right)(x) =$ $\sqrt{5/3}$

$\sqrt{\frac{4+1}{3}} = \sqrt{5/3}$

4. Using the given graph, find the following



a) $\lim_{x\to 2^+} f(x) =$ 3

b) $\lim_{x\to -3} f(x) =$ -2

c) $\lim_{x\to 2} f(x) =$ 0

d) $\lim_{x\to 0} f(x) =$ -1

e) $f(-3) =$ DNE

f) $\lim_{x\to 2^-} f(x) =$ 1

g) $f(0) =$ -1

h) $\lim_{x\to -3^+} f(x) =$ -2

i) $\lim_{x\to 1^-} f(x) =$ $+\infty$

6. Prove the following limit using the precise δ, ε definition of a limit.

$\lim_{x\to 2} (3 - x^2) = -1$

$\lim_{x\to a} f(x) = L$ if for any $\varepsilon > 0$ there exists a corresponding $\delta > 0$ so that $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|3 - x^2 - (-1)| < \varepsilon$ whenever $|x-2| < \delta$

$|4 - x^2| < \varepsilon$

$|2+x| \cdot |2-x| < \varepsilon$

$|x-2| < \varepsilon / |2+x|$

let $\delta = 1$

$|x-2| < 1$

$|x+2| < 5$

$|x-2| < \varepsilon/5$ so choose $\delta = \min\{1, \varepsilon/5\}$

check: $|x-2| < \delta \rightarrow |x-2| < \varepsilon/5 \rightarrow 5|x-2| < \varepsilon$

$|x+2| \cdot |x-2| < \varepsilon \rightarrow |x^2 - 4| < \varepsilon$ so $|4 - x^2| < \varepsilon$

$|3 - x^2 - (-1)| < \varepsilon$ ✓