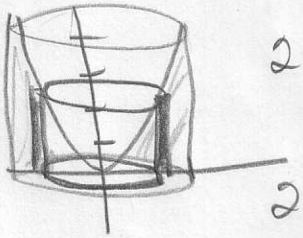
MATH 151 | **QUIZ 10A** | NAME Answers
Mrs. Bonny Tighe | 6.3-6.5 |
25 points | SECTION _______ Mon. 5/15/06

1. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line using the cylindrical shells method. Sketch the region.

$y = x^2$, $x = 0$ *and* $x = 2$, about the y-axis

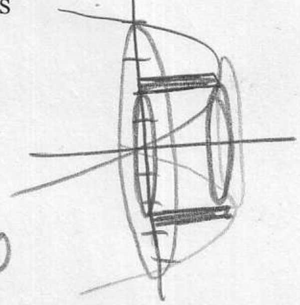


$$2\pi \int_0^2 x\, f(x)\, dx$$

$$2\pi \int_0^2 x\,(x^2)\,dx =$$

$$2\pi \frac{1}{4}x^4 \Big|_0^2 = \frac{\pi}{2}(2)^4 = 8\pi$$

b) $x = 4y - y^2$, $x = 0$; about the x -axis



$$\int_0^4 2\pi y\, f(y)\, dy$$

$$2\pi \int_0^4 y(4y - y^2)\,dy$$

$$2\pi \int_0^4 4y^2 - y^3\, dy =$$

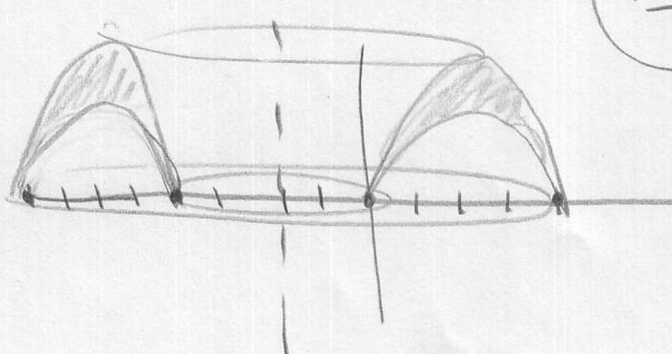
$$2\pi \left[\frac{4}{3}y^3 - \frac{1}{4}y^4\right]\Big|_0^4 =$$

$$2\pi \left(\frac{4}{3}(4)^3 - \frac{1}{4}(4)^4 - 0\right) =$$

$$2\pi (4^4)\left[\frac{1}{3} - \frac{1}{4}\right] = \frac{4^4 2\pi}{12}$$

$$\frac{128\pi}{3}$$

c) $y = 4x - x^2$, $y = 8x - 2x^2$ about x = - 2



$$4x - x^2 = 8x - 2x^2$$
$$x^2 - 4x = 0$$
$$x(x-4) = 0$$
$$x = 0, 4 \qquad (0,0) \quad (4,0)$$

$$2\pi \int_0^4 (x+2)\left[(8x - 2x^2) - (4x - x^2)\right]dx = 2\pi \int_0^4 (x+2)\left[4x - x^2\right]dx$$

$$2\pi \int_0^4 (2x^2 - x^3 + 8x)\,dx = 2\pi\left(\frac{2}{3}x^3 - \frac{1}{4}x^4 + 4x^2\right)\Big|_0^4 =$$

$$2\pi\left[\frac{2}{3}(4)^3 - \frac{1}{4}(4^4) + 4(4^2)\right] = 2\pi(4^3)\left[\frac{2}{3} - 1 + 1\right] = 128\pi\left(\frac{2}{3}\right) = \frac{256\pi}{3}$$

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

2. Find the average value of the function on the given interval:

a) $f(\theta)=\sin^4\theta\cos\theta$ on $[0,\pi/4]$

b) $g(x)=\dfrac{4}{(1+x)^2}$ on $[0,2]$

$$\frac{1}{\pi/4-0}\int_0^{\pi/4}\sin^4\theta\cos\theta\,d\theta$$

$u=\sin\theta$

$du=\cos\theta\,d\theta$

$$\frac{4}{\pi}\int u^4\,du$$

$$\frac{4}{\pi}\cdot\frac{1}{5}u^5\Big|_0^{\pi/4}=\frac{4}{5\pi}(\sin\theta)^5\Big|_0^{\pi/4}$$

$$\frac{4}{5\pi}(\sin\pi/4)^5-\frac{4}{5\pi}(\sin 0)^5=$$

$$\frac{4}{5\pi}\left(\frac{\sqrt{2}}{2}\right)^5$$

$$\frac{1}{2-0}(4)\int_0^2(1+x)^{-2}\,dx$$

$$\frac{4}{2}\left(\frac{1}{-1}(1+x)^{-1}\right)\Big|_0^2=$$

$$2\left(-\frac{1}{1+x}\right)\Big|_0^2=$$

$$\frac{-2}{1+2}-2\left(-\frac{1}{1}\right)=\frac{-2}{3}+1\quad 4/3$$

c) $y=x\cos(x^2)$ on $[0,\frac{\pi}{2}]$

$$\frac{1}{\pi/2-0}\int_0^{\pi/2}\cos x^2\,x\,dx$$

$$\left(\frac{2}{\pi}\right)\frac{1}{2}\int_0^{\pi/2}\cos u\,du$$

$$\frac{1}{\pi}(\sin u)\Big|_0^{\pi/2}$$

$$\frac{1}{\pi}\sin x^2\Big|_0^{\pi/2}=\frac{1}{\pi}\left[\sin(\pi/2)^2-\sin(0)^2\right)=$$

$$\boxed{\frac{1}{\pi}\sin(\pi/2)^2}$$