MATH 151
Mrs. Bonny Tighe

FINAL EXAM

200 points

NAME _____

SECTION_____Fri 5/19/06

There are 20 problems worth 10 points each.

1. Find the following limits.

a) $\lim_{x\to 0}\frac{\sin x}{2x} =$ _____

b) $\lim_{x\to 4}\frac{\sqrt{x}-1}{1-x} =$ _____

c) $\lim_{x\to \infty}\frac{2x-1}{3-x} =$ _____

2. Use the **definition of the derivative of a function** at a point as the limit of the slope of the secant as h approaches 0 to evaluate f '(x) for f(x) = $\frac{1}{x+1} - 2$

3. Differentiate: $f(x)= \dfrac{\sin^3 x}{1-x^2}$

4. Find dy/dx. $2x - 3\sec y = x^2 y^3$

5. Find the equation of the tangent line to $y=1+\cos^2 x$ *at* $(\frac{\pi}{6}, \frac{7}{4})$

6. Air is being pumped into a spherical balloon and the radius is expanding at the rate of $10\ cm\ /\min$. Find how fast the volume of the balloon is increasing when the radius is 20 cm. $V = \frac{4}{3}\pi r^3$

7. Find the asymptotes and intercepts. $f(x) = \frac{x^2 - 3x - 4}{x^2 - 4x}$

8. A box is to be constructed from 900 cm^2 of material with an open top and a square base. Find the maximum volume that can be contained in this box.

9. Find the intercepts, intervals of increasing, decreasing, concave up and concave down and sketch. $f(x) = x^4 - 8x^2 - 20$

10. Use the Fundamental Theorem of Calculus to evaluate the area under the curve $f(x) = \csc^2 x$ on the interval $[\pi/6, \pi/4]$.

11. A particle is moving with velocity, $v(t) = 4t - 4 \ m/s$. Find the displacement of the particle and then find the distance traveled on the interval [0,3].

12. Evaluate the definite integral, if it exists. $\int_1^2 \frac{\left(\frac{1}{x}+2\right)^3}{x^2} dx =$ _____

13. Evaluate the area under the curve f(x) = $2x^2 + 1$ on the interval [0,2], using the definition, as a limit of the summation as n approaches infinity and evaluate using the following:

$$\sum_{i=1}^{n} c = cn \qquad \sum_{i=1}^{n} i = \frac{n^2}{2} + \frac{n}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{2n^3}{6} + \frac{3n^2}{6} + \frac{n}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

14. Evaluate the indefinite integral using substitution .

$\int \sec^2(\sin 3x)\cos 3x \; dx =$ _____

15. Find the area bounded by the y-axis, the function $y = \sqrt{x-1}$, y=0 and y=2 Sketch the graph.

16. Find the points, if any, on the curve $g(x) = (x^2 - 1)^3$ where the tangent line is horizontal.

17. Find the second derivative for the function $h(x) = \tan 3x + 2x$

18. Sketch the region enclosed by the given curves, decide whether to integrate with respect to x or y. Find the area that is enclosed. $x = y^2$ and $y = 2 - x$

19. Rotate the region bounded by $y = x$ and $y = x^2$ about the line $y = -2$. Find the volume of the solid that has been generated using the washer or slicing method.

20. Use the method of cylindrical shells to find the volume generated by the rotation of the region bounded by $y = 3x - x^2$ and $y = x$ about the y-axis.

21. Find the average value of $f(x) = 4x - x^2$ on the interval $[0,3]$. Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f.

22. Use Newton's Method to approximate $\sqrt{35}$ to the third iteration, x_3.