MATH 151
Mrs. Bonny Tighe

EXAM III A
4.10-6.2
100 points

Name Answers

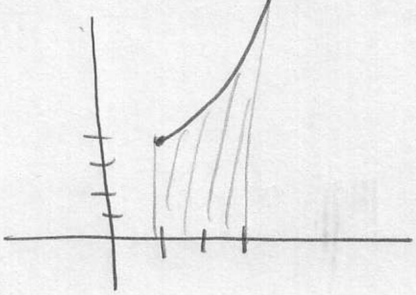
Section ______ 5/8/06

1. Find f(x). $f''(x) = \cos x + 2x - 1$, $f'(0)=1$ and $f(0)=4$

$$f'(x) = \sin x + x^2 - x + C \qquad f'(x) = \sin x + x^2 - x + 1$$
$$1 = 0 + 0 - 0 + C$$

$$f(x) = -\cos x + \tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 + x + C$$
$$4 = -1 + 0 - 0 + 0 + C \qquad \boxed{f(x) = -\cos x + \tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 + x + 5}$$
$$5 = C$$

2. Approximate the area under the curve $f(x) = 3x + x^2$ on the interval $1 \leq x \leq 3$ with four subintervals, n=4, taking the sample points to be the right endpoints, then the left endpoints.



$$\Delta x = \frac{3-1}{4} = \frac{1}{2}$$

1 $\quad$ 3/2 $\quad$ 2 $\quad$ 5/2 $\quad$ 3

$$A_R = \tfrac{1}{2}\left[f(\tfrac{3}{2}) + f(2) + f(\tfrac{5}{2}) + f(3)\right] = \tfrac{1}{2}\left[3(\tfrac{3}{2}) + \tfrac{9}{4} + 6 + 4 + \tfrac{15}{2} + \tfrac{25}{4} + 9 + 9\right]$$

$$\tfrac{1}{2}\left[\tfrac{9}{2} + \tfrac{9}{4} + 28 + \tfrac{15}{2} + \tfrac{25}{4}\right] = \tfrac{1}{2}\left[\tfrac{24}{2} + \tfrac{34}{4} + 28\right] = 6 + \tfrac{17}{4} + 14 = 20 + 4\tfrac{1}{4}$$

$$24\tfrac{1}{4}$$

$$A_L = \tfrac{1}{2}\left[f(1) + f(\tfrac{3}{2}) + f(2) + f(\tfrac{5}{2})\right] = \tfrac{1}{2}\left[3 + 1 + \tfrac{9}{2} + \tfrac{9}{4} + 6 + 4 + \tfrac{15}{2} + \tfrac{25}{4}\right]$$

$$\tfrac{1}{2}\left[14 + \tfrac{24}{2} + \tfrac{34}{4}\right] = 7 + 6 + \tfrac{17}{4} = 13 + 4\tfrac{1}{4} = 17\tfrac{1}{4}$$

Check: $3x + 2x^2 - \frac{1}{3}x^3 \Big|_2^4 = \left(3(4) + 2(16) - \frac{1}{3}(64)\right) - \left(6 + 8 - \frac{8}{3}\right)$

$12 + 32 - \frac{64}{3} - 14 + 8/3 = 44 - 14 - 56/3$

$30 - 18\frac{2}{3} = 11\frac{1}{3}$ ✓

3. Use the limit of sums definition of integration to evaluate the area given by

$\int_2^4 (3+4x-x^2)\,dx$ using $\sum_{i=1}^{n} c = cn$ $\quad \sum_{i=1}^{n} i = \frac{n^2}{2} + \frac{n}{2}$ $\quad \sum_{i=1}^{n} i^2 = \frac{2n^3}{6} + \frac{3n^2}{6} + \frac{n}{6}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(3 + 4\left(2 + \frac{2i}{n}\right) - \left(2 + \frac{2i}{n}\right)^2\right) =$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left(3 + 8 + \frac{8i}{n} - 4 - \frac{8i}{n} - \frac{4i^2}{n^2}\right) = \lim_{n\to\infty} \sum_{i=1}^{n} \left(7 - \frac{4i^2}{n^2}\right)\frac{2}{n}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{14}{n} - \frac{8i^2}{n^3}\right) = \lim_{n\to\infty} \left(\frac{14}{n}(n) - \frac{8}{n^3}\left(\frac{2n^3}{6} + \frac{3n^2}{6} + \frac{n}{6}\right)\right)$$

$$\lim_{n\to\infty} \left(14 - \frac{8}{3} - \frac{24}{6n} - \frac{8}{6n^2}\right) = 14 - 8/3 = 14 - 2\tfrac{2}{3} = 11\tfrac{1}{3}$$

4. A particle moves along a line with the velocity function $v(t) = 2 - t$. Find the total distance traveled by the particle during the time interval [0,4].

$v = 0 \quad 2 - t = 0 \quad t = 2\text{ sec}$

$$\int_0^2 (2-t)\,dt + \int_2^4 (2-t)\,dt$$

$$2t - \frac{1}{2}t^2 \Big|_0^2 + 2t - \frac{1}{2}t^2 \Big|_2^4$$

$$\left(4 - \frac{1}{2}(4)\right) - (0) + \left(8 - \frac{1}{2}(16)\right) - \left(4 - \frac{1}{2}(4)\right)$$

$$2 + |-(2)| = 4$$

5. Evaluate the indefinite integral, if it exists

a) $\int x\sqrt{2x^2+3}\,dx =$ ________________ b) $\int \frac{\sin 2\theta}{\cos^4\theta}\,d\theta =$ ________________

$2\sin\theta\cos\theta$

$\frac{1}{4}\int u^{1/2}\,du$ $u = 2x^2+3$

$du = 4x\,dx$

$\frac{1}{4}\cdot\frac{1}{3/2}u^{3/2}+C$

$\frac{1}{4}\cdot\frac{2}{3}(2x^2+3)^{3/2}+C$

$\frac{1}{6}(2x^2+3)^{3/2}+C$

$-2\int \cos^{-3}\theta(-\sin\theta\,d\theta)$

$-2\int u^{-3}\,du$ $u = \cos\theta$

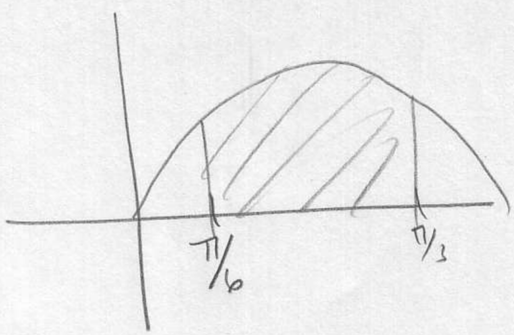
$du = -\sin\theta\,d\theta$

$-2\cdot\frac{1}{-2}u^{-2}+C$

$\frac{1}{\cos^2\theta}+C = \sec^2\theta + C$

or $\tan^2\theta + C$

6. Find the area bounded by the x-axis and the function on the given interval. Sketch the graph. $f(x) = \sin 2x$ *on the* int*erval* $[\pi/6, \pi/3]$



$\int_{\pi/6}^{\pi/3} \sin 2x\,dx$

$-\frac{1}{2}\cos 2x\,\Big|_{\pi/6}^{\pi/3}$

$-\frac{1}{2}\cos 2\pi/3 - \left(-\frac{1}{2}\cos\pi/3\right)$

$-\frac{1}{2}\left(-\frac{1}{2}\right) + \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{2}$

$\frac{1}{4} + \frac{1}{4} =$

7. a) Give the definite integral defined by $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{4}{n}\left[3\frac{4i}{n}-\frac{n}{4i}+\tan\frac{8i}{n}\right]$

$\int_0^4 (3x - \frac{1}{x} + \tan 2x)\,dx$

b) The population of a new housing development starts with 10 occupied homes and increases at a rate of $h'(t)$ per month. What does $10+\int_0^{24} h'(t)\,dt$ represent?

The total change in the population of the housing development for the first 2 years.

8. Sketch the region enclosed by the given curves, decide whether to integrate with respect to x or y, and find the area bounded by the two graphs. Sketch the graph.

$x = 4y - y^2$ *and* $x = y^2 - 2y$

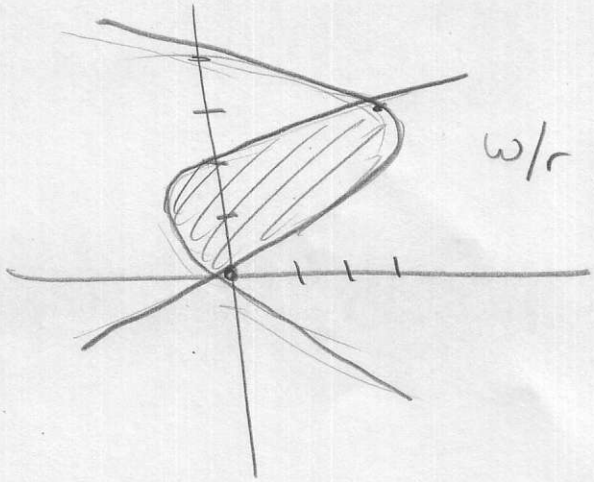
$4y - y^2 = y^2 - 2y$

$-2y^2 + 6y = 0$

$-2y(y-3) = 0$

$y = 0, 3$

$x = 12 - 9$



w/r to y-axis

$\int_0^3 (4y - y^2) - (y^2 - 2y)\,dy$

$\int_0^3 (6y - 2y^2)\,dy$

$3y^2 - \frac{2}{3}y^3 \Big|_0^3 = \left(3(3)^2 - \frac{2}{3}(3)^3\right) - (0 - 0)$

$27 - \frac{54}{3} = 27 - 18 = 9$

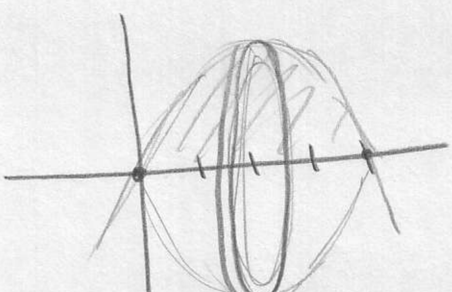
9. Evaluate the definite integral, it if exists. $\int_1^2 \frac{x^3+1}{x^2}dx =$ ___2___

$$\int_1^2 x + x^{-2}\, dx = \frac{1}{2}x^2 + \frac{1}{-1}x^{-1}\Big|_1^2 =$$

$$\left(\frac{1}{2}(4) - \frac{1}{2}\right) - \left(\frac{1}{2} - \frac{1}{1}\right)$$

$$2 - \frac{1}{2} + \frac{1}{2} = 2$$

10. Find the volume of the solid obtained by rotating the region bounded by $y = 4x - x^2$ *and the* $x - axis$ about the x-axis. Sketch the region and a typical disc.



$$\int_0^4 \pi r^2 dx =$$

$$\pi \int_0^4 (4x-x^2)^2 dx$$

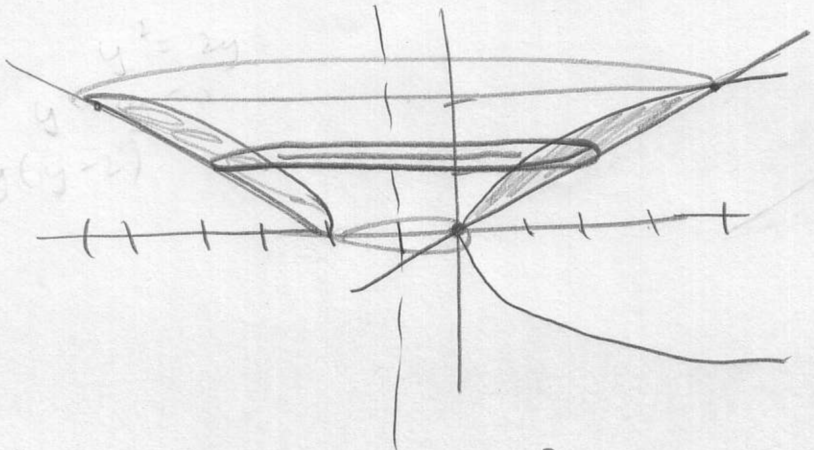
$$\pi \int_0^4 (16x^2 - 8x^3 + x^4)\, dx = \pi\left(\frac{16}{3}x^3 - \frac{8}{4}x^4 + \frac{1}{5}x^5\right)\Big|_0^4 =$$

$$\pi\left(\frac{16}{3}(4)^3 - 2(4)^4 + \frac{1}{5}(4^5)\right) = (4)^5\pi\left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right) = \pi 4^5\left(\frac{10-15+6}{30}\right)$$

$$\frac{4^5\pi}{30}$$

11. Find the volume of the solid obtained by rotation the region bounded by the two given curves about the line x = - 1. Sketch the region and a typical washer.

$y^2 = x \text{ and } x = 2y$



$y^2 = 2y$

$y^2 - 2y = 0$

$y(y-2) = 0$

$(0,0) \quad (4,2)$

$y^2 = x + 1$

$x + 1 = 2y$

$x = 2y - 1$

$\pi \int_0^2 (2y+1)^2 - (y^2+1)^2 \, dy$

$\pi \int_0^2 (4y^2 + 4y + 1) - (y^4 + 2y^2 + 1) \, dy$

$\pi \int_0^2 (2y^2 + 4y - y^4) \, dy = \pi \left(\frac{2}{3} y^3 + \frac{4}{2} y^2 - \frac{1}{5} y^4 \right) \Big|_0^2$

$\pi \left(\frac{2}{3} y^3 + 2y^2 - \frac{1}{5} y^4 \right) \Big|_0^2 = \pi \left(\frac{2}{3}(8) + 2(4) - \frac{1}{5}(16) \right) - (0)$

$\pi \left(\frac{16}{3} + 8 - \frac{16}{5} \right) = \pi \left(\frac{16+24}{3} - \frac{16}{5} \right) = \pi \left(\frac{40}{3} - \frac{16}{5} \right) = \frac{152\pi}{15}$

$\pi \left(\frac{200-48}{15} \right)$