MATH 151
Mrs. Bonny Tighe

EXAM III

4.10-6.2
100 points

Name Answers

Section ______ 5/8/06

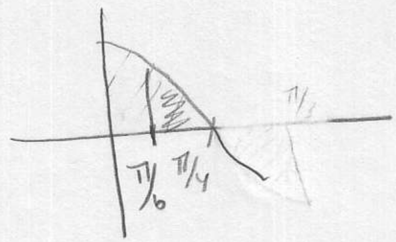
1. Find the area bounded by the x-axis and the function on the given interval. Sketch the graph. $f(x) = \cos 2x$ *on the* int *erval* $[\pi/6, \pi/\cancel{3}4]$

$$\int_{\pi/6}^{\pi/4} \cos 2x \, dx =$$

$$+\frac{1}{2}\sin 2x \Big|_{\pi/6}^{\pi/4} =$$

$$\left(-\frac{1}{2}\sin \pi/2\right) - \left(+\frac{1}{2}\sin \pi/3\right) =$$

$$+\frac{1}{2}(1) + \frac{1}{2}\left(\frac{-\sqrt{3}}{2}\right) = +\frac{1}{2} + \frac{-\sqrt{3}}{4}$$



2. a) Give the definite integral defined by $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left[3\frac{2i}{n} - \frac{n}{2i} + \sin\frac{4i}{n}\right]$

$$\int_0^2 \left(3x - \frac{1}{x} + \sin 2x\right) dx$$

b) The population of a new housing development starts with 5 occupied homes and increases at a rate of $h'(t)$ per month. What does $5 + \int_0^{18} h'(t)\,dt$ represent?

The total change in the population of the housing development for the first 18 months

3. Use the limit of sums definition of integration to evaluate the area given by

$\int_1^4 (4+2x-x^2)dx$ using $\sum_{i=1}^{n} c = cn$ $\sum_{i=1}^{n} i = \frac{n^2}{2}+\frac{n}{2}$ $\sum_{i=1}^{n} i^2 = \frac{2n^3}{6}+\frac{3n^2}{6}+\frac{n}{6}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left(4+2\left(1+\frac{3i}{n}\right)-\left(1+\frac{3i}{n}\right)^2\right) =$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left(4+2+\frac{6i}{n}-1-\frac{6i}{n}-\frac{9i^2}{n^2}\right) = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left(5-\frac{9i^2}{n^2}\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n}\left(\frac{15}{n}-\frac{27i^2}{n^3}\right) = \lim_{n\to\infty}\left(\frac{15}{n}(n)-\frac{27}{n^3}\left(\frac{2n^3}{6}+\frac{3n^2}{6}+\frac{n}{6}\right)\right) =$$

$$\lim_{n\to\infty}\left(15-\frac{27}{3}-\frac{27(3)}{6n}-\frac{27n}{6n^2}\right) = 15-9 = 6$$

Check $4x+x^2-\frac{1}{3}x^3\Big|_1^4 = \left(16+16-\frac{64}{3}\right)-\left(4+1-\frac{1}{3}\right)$

$32-\frac{64}{3}-5+\frac{1}{3}$ $\quad 27-\frac{63}{3} = 27-21 = 6$ ✓

4. A particle moves along a line with the velocity function $v(t) = 3-t$. Find the total distance traveled by the particle during the time interval [0,5].

$v(t)=0$ $\quad t=3$ the particle turns around

$$\left|\int_0^3 (3-t)\,dt\right| + \left|\int_3^5 (3-t)\,dt\right|$$

$$3t-\frac{1}{2}t^2\Big|_0^3 + 3t-\frac{1}{2}t^2\Big|_3^5$$

$$\left(9-\frac{9}{2}\right)-(0) + \left(15-\frac{25}{2}\right)-\left(9-\frac{9}{2}\right)$$

$$\left|\frac{9}{2}\right| + \left|15-\frac{25}{2}-\frac{9}{2}\right|$$

$|-2|$ 24

$4\frac{1}{2}+2 = 6\frac{1}{2}$

5. Find f(x). $f''(x) = \sin x + 4x - 1$, $f'(0)=1$ and $f(0)=4$

$$f'(x) = -\cos x + 2x^2 - x + C \rightarrow f'(x) = -\cos x + 2x^2 - x + 2$$

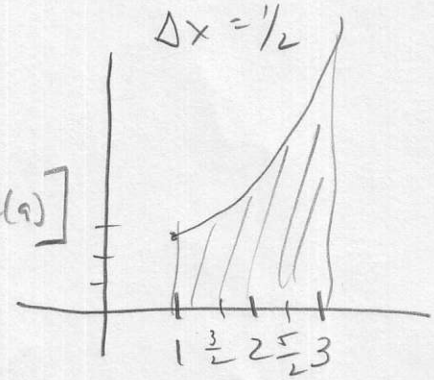
$$1 = -\cos 0 + 0 - 0 + C \qquad f(x) = -\sin x + \tfrac{2}{3}x^3 - \tfrac{1}{2}x^2 + 2x + C$$

$$2 = C \qquad 4 = C$$

$$f(x) = -\sin x + \tfrac{2}{3}x^3 - \tfrac{1}{2}x^2 + 2x + 4$$

6. Approximate the area under the curve $f(x) = x + 2x^2$ on the interval $1 \le x \le 3$ with four subintervals, n = 4, taking the sample points to be the right endpoints, then the left endpoints.

$\Delta x = 1/2$



$$A_R = \tfrac{1}{2}\left[f(3/2) + f(2) + f(5/2) + f(3)\right]$$

$$= \tfrac{1}{2}\left[\tfrac{3}{2} + 2\left(\tfrac{9}{4}\right) + 2 + 2(4) + \tfrac{5}{2} + 2\left(\tfrac{25}{4}\right) + 3 + 2(9)\right]$$

$$= \tfrac{1}{2}\left[\tfrac{3}{2} + \tfrac{9}{2} + 10 + \tfrac{5}{2} + \tfrac{25}{2} + 21\right] = \tfrac{1}{2}\left[31 + \tfrac{42}{2}\right] =$$

$$\tfrac{1}{2}[31 + 21] = \tfrac{1}{2}[52] = 26$$

90
12

$$A_L = \tfrac{1}{2}\left[f(1) + f(3/2) + f(2) + f(5/2)\right]$$

$$\tfrac{1}{2}\left[1 + 2 + \tfrac{3}{2} + 2\left(\tfrac{9}{4}\right) + 2 + 2(4) + \tfrac{5}{2} + 2\left(\tfrac{25}{4}\right)\right] =$$

$$\tfrac{1}{2}\left[3 + \tfrac{3}{2} + \tfrac{9}{2} + 10 + \tfrac{5}{2} + \tfrac{25}{2}\right] = \tfrac{1}{2}\left[13 + \tfrac{42}{2}\right] = \tfrac{1}{2}[13 + 21] =$$

$$\tfrac{1}{2}[34] = 17$$

7. Evaluate the indefinite integral, if it exists

a) $\int x^2 \sqrt{x^3+4}\, dx =$ ______________ b) $\int \tan^2 \theta \, d\theta =$ ____________

$\frac{1}{3} \int u^{1/2}\, du$ $u = x^3+4$ $du = 3x^2\, dx$

$\frac{1}{3} \cdot \frac{1}{3/2} u^{3/2} + C =$

$\frac{1}{3} \cdot \frac{2}{3} (x^3+4)^{3/2} + C =$

$\frac{2}{9}(x^3+4)^{3/2} + C$

$\int \sec^2\theta - 1 \, d\theta =$

$\tan\theta - \theta + C$

8. Sketch the region enclosed by the given curves, decide whether to integrate with respect to x or y, and find the area bounded by the two graphs. Sketch the graph.

$x = 6y - y^2$ *and* $x = y^2 - 2y$

$6y - y^2 = y^2 - 2y$

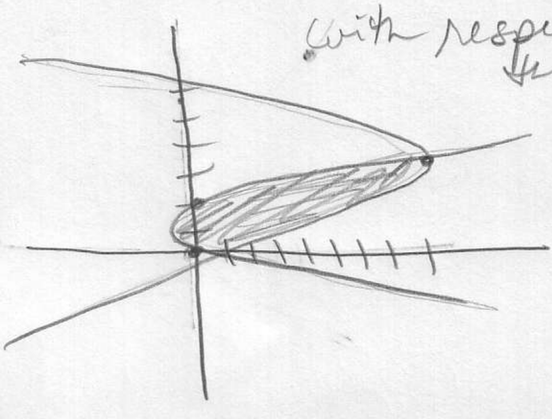
$2y^2 - 8y = 0$

$2y(y-4) = 0$

$y = 0$ and 4

$(0,0)$ $(8,4)$

with respect to the y-axis



$\int_0^4 \left((6y-y^2) - (y^2-2y)\right) dy = \int_0^4 (8y - 2y^2)\, dy = 4y^2 - \frac{2}{3}y^3 \Big|_0^4 =$

$\left(4(16) - \frac{2}{3}(64)\right) - (0) = 64 - \frac{128}{3} = \frac{3(64)}{3} - \frac{2(64)}{3} = \frac{64}{3} = 21\frac{1}{3}$

9. Evaluate the definite integral, it if exists. $\int_1^2 \frac{x^4+3}{x^2}dx =$ ____________

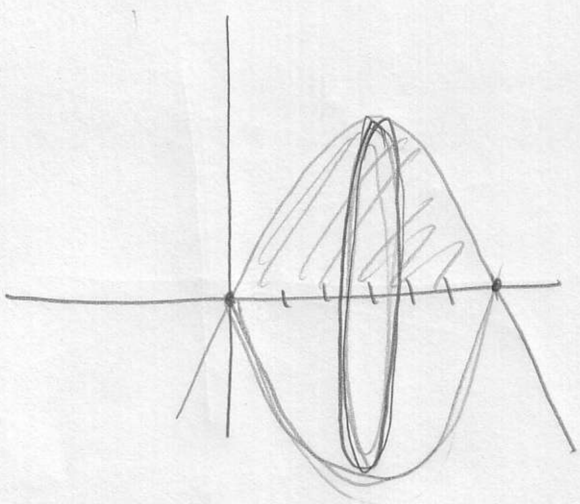
$\int_1^2 x^2 + 3x^{-2}\,dx = \frac{1}{3}x^3 + 3\cdot\frac{1}{-1}x^{-1}\Big|_1^2 =$

$\frac{1}{3}x^3 - \frac{3}{x}\Big|_1^2 = \left(\frac{1}{3}(8) - \frac{3}{2}\right) - \left(\frac{1}{3} - 3\right) = \frac{8}{3} - \frac{3}{2} - \frac{1}{3} + 3$

$\frac{2}{2}\cdot\frac{7}{3} + \frac{3}{2}\cdot\frac{3}{3}$

$\frac{14}{6} + \frac{9}{6} = 23/6$

10. Find the volume of the solid obtained by rotating the region bounded by $y = 6x - x^2$ *and the* $x - axis$ about the x-axis. Sketch the region and a typical disc.



$\int \pi r^2 dx$

$\pi\int_0^6 (6x - x^2)^2 dx$

$\pi\int_0^6 (36x^2 - 12x^3 + x^4)\,dx =$

$\pi\left[\frac{36}{3}x^3 - \frac{12}{4}x^4 + \frac{1}{5}x^5\right]\Big|_0^6 =$

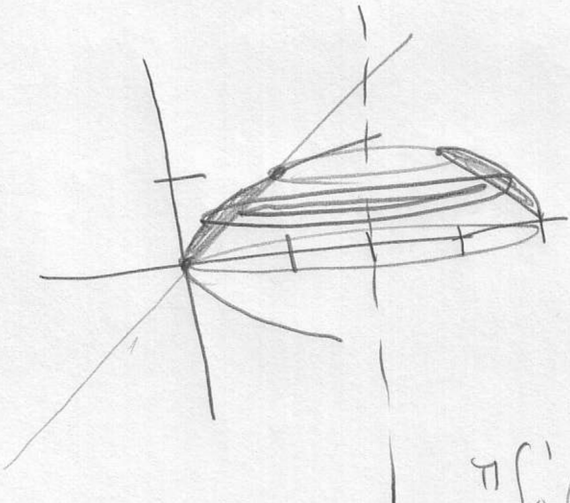
$\pi\left(12x^3 - 3x^4 + \frac{1}{5}x^5\right)\Big|_0^6 =$

$\pi\left(12(6^3) - 3(6)^4 + \frac{1}{5}(6^5)\right) - 0$

$\pi 6^4\left[2 - 3 + 6/5\right] = 6^4\pi\left(\frac{1}{5}\right) = \frac{\pi 6^4}{5}$

11. Find the volume of the solid obtained by rotation the region bounded by the two given curves about the line x = 2. Sketch the region and a typical washer.

$y^2 = x$ *and* $x = y$



$y^2 = x + 2$ $\qquad$ $y = x + 2$

$y^2 - 2 = x$ $\qquad$ $y - 2 = x$

$$\pi \int_0^1 (y^2-2)^2 - (y-2)^2 \, dy$$

$$\pi \int_0^1 (y^4 - 4y^2 + \cancel{4} - y^2 + 4y - \cancel{4}) \, dy$$

$$\pi \int_0^1 (y^4 - 5y^2 + 4y) \, dy = \pi \left(\tfrac{1}{5} y^5 - \tfrac{5}{3} y^3 + 2y^2 \right) \Big|_0^1$$

$$\pi \left(\tfrac{1}{5} - \tfrac{5}{3} + 2 \right) - \pi(0)$$

$$\pi \left(\left(\tfrac{3}{3}\right)\tfrac{11}{15} - \tfrac{5}{3}\left(\tfrac{5}{5}\right) \right) = \frac{\pi 8}{15}$$