MATH 151 **EXAM IIA** NAME Answers

Mrs. Bonny Tighe 3.9-4.9

100 points SECTION _____ Mon. 4/10/06

There are 11problems worth 10 points each.

1. Find the absolute and local maximum and minimum values of the function f(x) on the given interval. $f(x) = 3 + \sin 2x \quad [0, \pi/6]$

$f(0) = 3$ absolute minimum

$f(\pi/6) = 3 + \sin \pi/3 =$

$3 + \sqrt{3}/2$ absolute maximum

$f'(x) = \cos 2x(2) = 0$

$2x = \pi/2$

$x = \pi/4$

not on interval

2. Find the limits:

a) $\lim_{x\to-\infty} = \dfrac{2x-3}{4-x^2}$ $\underline{\quad 0 \quad}$

b) $\lim_{x\to\infty}(2x - \sqrt{4x^2+5}) = \underline{\quad 0 \quad}$

$(2x - \sqrt{4x^2+5})\left(\dfrac{2x+\sqrt{4x^2+5}}{2x+\sqrt{4x^2+5}}\right)$

$\dfrac{\cancel{4x^2} - \cancel{4x^2} - 5}{2x + \sqrt{4x^2+5}}$

c) $\lim_{x\to\infty} \dfrac{\sqrt{4x^2+3x-1}}{5x+3} = \underline{\quad 2/5 \quad}$

d) $\lim_{x\to-\infty} (2-x)^3(3x+1)(2x-3) = \underline{+\infty}$

$-x^5$

3. Find the vertical and horizontal asymptotes as well as the x- and y-intercepts:

a) $f(x) = \dfrac{x^3 - x}{4 - x^2}$ $\quad x(x^2-1)$

x-int: (0,0) (1,0) (-1,0)

y-int: (0,0)

VA: $x = \pm 2$

Slant: $y = -x$

$$\begin{array}{r} -x \\ -x^2+4 \,\big)\, x^3 - x \\ -(-x^3 - 4x) \\ \hline 3x \end{array}$$

b) $f(x) = \dfrac{x+1}{x^2 - 4x + 3}$

$(x-3)(x-1)$

x-int: (-1,0)

y-int: (0, 1/3)

VA: x = 3, 1

HA: y = 0

4. Find the equation of the tangent line to y = $2 - \cos^3 3x$ at $(\pi/6, 2)$

$y - y_1 = m(x - x_1)$

$y - 2 = (0)(x - \pi/6)$

$y = 2$

$m = dy/dx =$

$-[3\cos^2 3x(-\sin 3x)(3)]$

$9\cos^2 \pi/2 \sin \pi/2 = 0$

5. Use Linear Approximation, or differentials, to estimate the value of sin 61°

$L(x) = f(a) + f'(a)(x - a)$

$= \sqrt{3}/2 + (1/2)(1°)\frac{\pi}{180}$

$\sqrt{3}/2 + \frac{\pi}{360}$

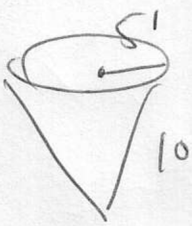
$f(x) = \sin x$

$f'(x) = \cos x$

$f(a) = \sin 60° = \sqrt{3}/2$

$f'(a) = \cos 60° = 1/2$

6. A conical tank is 10 feet high and has a radius of 5 feet. If the tank is being filled with water and the water level is rising at a rate of 4 ft/min., how fast is the water being pumped into the tank when the water is 18 inches deep?



$r = \frac{1}{2}h$

Given: $dh/dt = 4$ ft/min

Find dV/dt

when $h = 1\frac{1}{2}$ ft.

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h$

$V = \frac{\pi}{12}h^3$

$dV/dt = \frac{\pi}{4}h^2\, dh/dt$

$dV/dt = \pi/4\left(\frac{3}{2}\right)^2(4)$

$\boxed{dV/dt = \frac{9\pi}{4}\ ft^3/min}$

7. Find the intervals of increasing and decreasing using the first derivative test. Find the intervals of concave up and concave down using the second derivative test. Find the intercepts and graph.

$f(x) = 2x^3 - 3x^2 - 12x + 18$

x-int:
$(\pm\sqrt{6}, 0)$
$(3/2, 0)$
y-int:
$(0, 18)$

$x^2(2x-3) - 6(2x-3)$

$(x^2-6)(2x-3)$

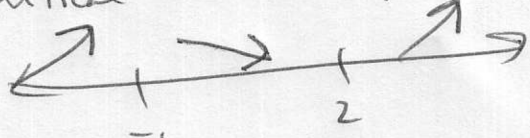
$x = \pm\sqrt{6}\quad x = 3/2$

$f'(x) = 6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2)$

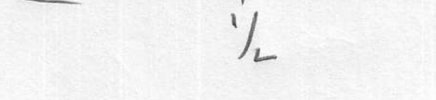
$(x-2)(x+1) = 0$

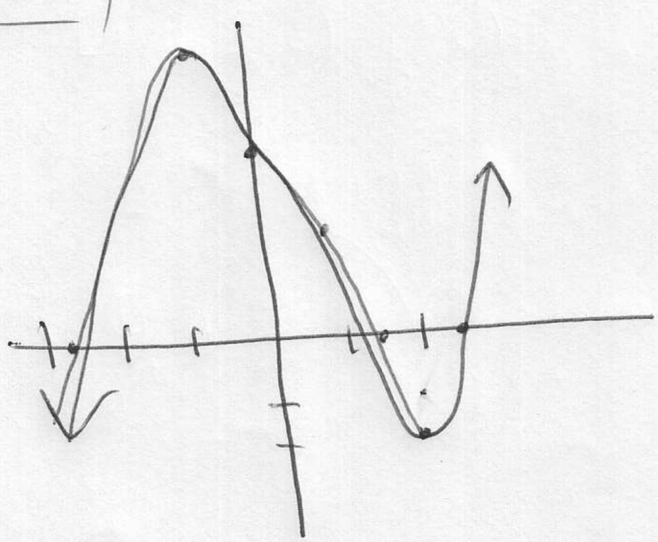
critical #s $x = 2, -1$



$f''(x) = 12x - 6 = 0$

$x = 1/2$ inflection point





$f(-1) = -2 - 3 + 12 + 18 = 25$

$f(2) = 16 - 12 - 24 + 18 = -2$

$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{8}\right) - 3\left(\frac{1}{4}\right) - 6 + 18$

$\frac{1}{4} - \frac{3}{4} + 12 = 11\frac{1}{2}$

8. Verify the hypotheses of the Mean Value Theorem and find all numbers c that satisfy the conclusion.

$f(x) = (4-x)\sqrt{x+1}$, $\cancel{[0,3]}$ $[-1,4]$

$f(x)$ is continuous on $[-1,4]$ ✓

$f(x)$ is differentiable on $(-1,4)$ so there must be a c on $(-1,4)$

so that $f'(c) = \dfrac{f(4)-f(-1)}{4-(-1)} = \dfrac{0-0}{5} = 0$

$f(4)=0$

$f(-1)=0$

$f'(x) = (4-x)\frac{1}{2}(x+1)^{-1/2} + (x+1)^{1/2}(-1) = 0$

$\dfrac{4-x}{2\sqrt{x+1}} - 1\sqrt{x+1} = 0$

$4-x-2(x+1) = 0$

$4-x-2x-2$

$-3x+2=0 \qquad x = 2/3$

so $c = 2/3$

9. Show that the equation $x^3 + 15x + 4 = 0$ has at most one root

$f(x) = x^3 - 15x + 4$ is a third degree equation with a positive leading coefficient so it goes from $-\infty$ to $+\infty$ and so there is at least one root.

$f'(x) = 3x^2 + 15 \neq 0$ There are no critical numbers so $f(x)$ cannot change direction, so there is only one root.

10. Use Newton's Method to approximate $\sqrt[3]{50}$

$x^3 - 50 = 0$

$x_1 = 3$ or 4

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_2 = 3 - \frac{27-50}{3(9)} = 3\tfrac{23}{27}$$

OR

$$x_2 = 4 - \frac{64-50}{3(64)} = 4 - \frac{14}{192}$$

$3\tfrac{178}{192}$

$$x_3 = 3\tfrac{23}{27} - \frac{(3\tfrac{23}{27})^3 - 50}{3(3\tfrac{23}{27})^2}$$

OR

$$x_3 = 3\tfrac{178}{192} - \frac{(3\tfrac{178}{192})^3 - 50}{3(3\tfrac{178}{192})^2}$$

11. If 1200 cm^2 of material is available to construct a box with a square base and an open top, find the largest possible volume of the box.

Objective

$V = lwh$

$V = x^2h$

$V = x^2\left(\frac{300}{x} - \frac{1}{4}x\right)$

$= 300x - \frac{1}{4}x^3$

$V' = 300 - \frac{3}{4}x^2 = 0$

$300\left(\frac{4}{3}\right) = x^2$

$400 = x^2$

$x = 20 \text{ cm}$

Constraint

Material $= 1200$

$4xh + x^2 = 1200$

$h = \frac{1200 - x^2}{4x}$

$h = \frac{300}{x} - \frac{1}{4}x$

$V = 20 \times 20 \times 10 \text{ cm}^3$