MATH 151 **EXAM II** NAME Answers

Mrs. Bonny Tighe 3.9-4.9

100 points SECTION _____ Mon. 4/10/06

There are 11problems worth 10 points each.

1. Find the equation of the tangent line to $y = 2 - \cos^3 2x$ at $(\pi/4, 2)$

$y - y_1 = m(x - x_1)$

$m = dy/dx = -3\cos^2 2x(-\sin 2x)(2)$

$= 6\cos^2 2x \sin 2x$

at $x = \pi/4$

$6(\cos \pi/2)^2(\sin \pi/2)$

0

$y - 2 = 0(x - \pi/4)$

$y = 2$

2. A conical tank is 6 meters high and has a radius of 2 m. If the tank is being filled with water and the water level is rising at a rate of 1 m/min., how fast is the water being pumped into the tank when the water is ~~50 cm~~ deep?

.5m

Find: dV/dt =

Given: $dh/dt = 1$ m/min

when $h = .5$m

$r = \frac{1}{3}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(\frac{1}{3}h)^2 h$

$V = \frac{\pi}{27}h^3$

$dV/dt = \frac{\pi}{9}h^2\, dh/dt$

$dV/dt = \frac{\pi}{9}(\frac{1}{2})^2(1) =$

$\frac{dV}{dt}$ $\frac{\pi}{36}$ m^3/min

3. Find the vertical and horizontal asymptotes as well as the x- and y-intercepts:

a) $f(x)=\dfrac{x^3+x}{9-x^2}$

X-int: $x=0$ $(0,0)$ $x(x^2+1)=0$

y-int: $(0,0)$

VA: $x=\pm 3$

Slant. $y=-x$

$$\begin{array}{r|l} & -x \\ -x^2+9 & x^3+x \\ & -(-x^3-9x) \\ & 10x \end{array}$$

b) $f(x)=\dfrac{2x+1}{x^2-4x+3}$

$(x-3)(x-1)$

x-int: $(-1/2, 0)$

y-int: $(0, 1/3)$

VA: $x=3, 1$

HA: $y=0$

4. Use Linear Approximation, or differentials, to estimate the value of $\tan 31^0$

$f(x)=\tan x$

$f'(x)=\sec^2 x$

$f(\pi/6)=\sqrt{3}/3$

$f'(\pi/3)=\left(\dfrac{2}{\sqrt{3}}\right)^2=\dfrac{4}{3}$

$L(x)=f(a)+f'(a)(x-a)$

$\dfrac{\sqrt{3}}{3}+\dfrac{4}{3}\left(\dfrac{\pi}{180}\right)$

$\dfrac{\sqrt{3}}{3}+\dfrac{\pi}{135}$

5. Use Newton's Method to approximate $\sqrt[3]{29}$

$x=\sqrt[3]{29}$ $\quad x^3=29$ $\quad x^3-29=0$ $\quad x_1=3$

$x_{n+1}=x_n-\dfrac{f(x_n)}{f'(x_n)}$

$f(x)=x^3-29$

$f'(x)=3x^2$

$x_2=3-\dfrac{27-29}{3(9)}=3+\dfrac{2}{18}=3\tfrac{1}{9}=28/9$

$x_3=28/9-\dfrac{(28/9)^3-29}{3(28/9)^2}$

6. Find the limits:

a) $\lim\limits_{x\to-\infty} = \dfrac{2x^2-3}{4-x^2}$ -2

b) $\lim\limits_{x\to\infty}(\sqrt{4x^2+5}-2x) =$ 0

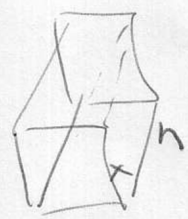
$$\sqrt{4x^2+5}-2x\left(\frac{\sqrt{4x^2+5}+2x}{\sqrt{4x^2+5}+2x}\right)=$$

$$\frac{\cancel{4x^2}+5-\cancel{4x^2}}{\sqrt{4x^2+5}+2x}=0$$

c) $\lim\limits_{x\to\infty}\dfrac{\sqrt{4x\ \ +3x-1}}{5x+3}=$ 0

d) $\lim\limits_{x\to\infty}(2-x)^2(3x+1)^2(2x-3)=$ $-\infty$

$+x^5$



7. If 1200 cm^2 of material is available to construct a box with a square base and an open top, find the largest possible volume of the box.

Objective:

$V = lwh$

$V = x^2h = x^2\left(\frac{300}{x}-\frac{1}{4}x\right)$

$= 300x - \frac{1}{4}x^3$

$V' = 300 - \frac{3}{4}x^2 = 0$

$x^2 = 300\left(\frac{4}{3}\right)$

400

$x = 20\,cm$

Constraint

$SA = x^2+4xh$

$1200 = x^2+4xh$

$\frac{1200-x^2}{4x} = h$

$\frac{300}{x}-\frac{1}{4}x = h$

$h = 10\,cm$

$V = (20\times 20\times 10)\ cm^3$

$2000\,cm^3$

8. Find the absolute and local maximum and minimum values of the function f(x) on the given interval. $f(x) = 3 + \sin 2x$ $[0, \pi/6]$

$f(0) = 3$ min

$f(\pi/6) = 3 + \sin \pi/3$

$3 + \sqrt{3}/2$ max

$f($

$f'(x) = (\cos 2x)(2) = 0$

$x = \pi/4$ not on interval

9. Find the intervals of increasing and decreasing using the first derivative test. Find the intervals of concave up and concave down using the second derivative test. Find the intercepts and graph.

$f(x) = 2x^3 - 12x^2 + 18x - 108$

x-int: $(6, 0)$

y-int: $(0, -108)$

no asymptotes

$2x^2(x-6) + 18(x-6) = 0$

$(2x^2+18)(x-6) = 0$ $x = 6$

$f(1) = -100$

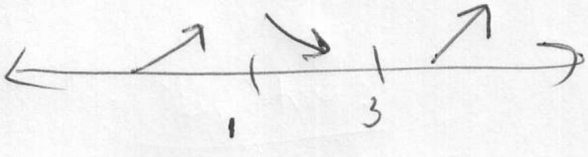
$f(3) = -108$

$f(2) = -104$

$f'(x) = 6x^2 - 24x + 18 = 0$

$6(x^2 - 4x + 3) = 0$

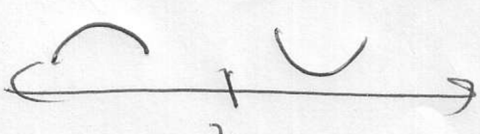
$6(x-3)(x-1)$ critical #'s

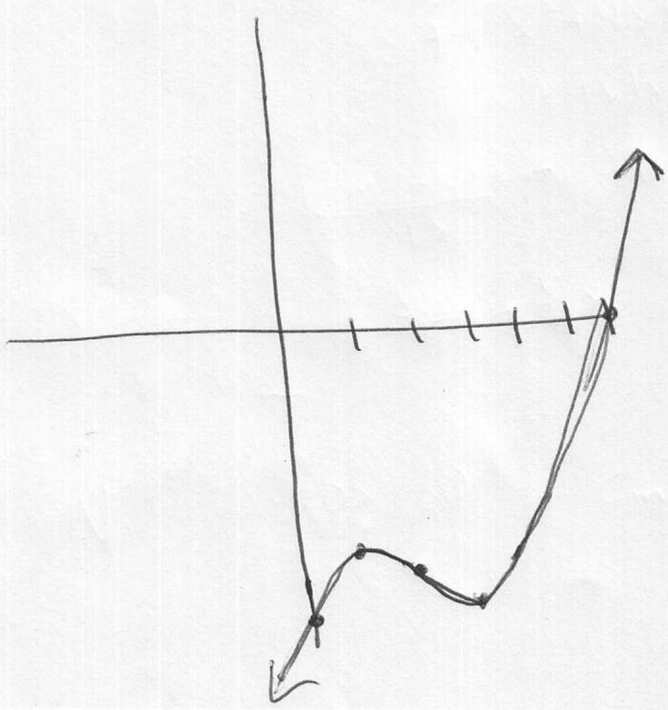


$f''(x) = 12x - 24 = 0$

$x = 2$

inflection pt.





10. Show that the equation $x^3 + 18x + 8 = 0$ has at most one root.

Because it is a third degree equation with a positive leading coefficient it will go from $-\infty$ to $+\infty$ and will have at least one root.

$f'(x) = 3x^2 + 18 \neq 0$ there are no critical numbers, so the function can not change direction so there is at most one root.

11. Verify the hypotheses of the Mean Value Theorem and find all numbers c that satisfy the conclusion.

$f(x) = (4-x)\sqrt{x+1}$, ~~[0,3]~~ $[-1,4]$

$f(x)$ is continuous on $[-1,4]$ ✓

$f(x)$ is differentiable on $(-1,4)$ ✓

So there must be a number c on $(-1,4)$ so that

$$f'(c) = \frac{f(4)-f(-1)}{4-(-1)} = \frac{0-0}{5}$$

$f(4) = 0(\sqrt{5}) = 0$

$f(-1) = 5\sqrt{0} = 0$

$$f'(x) = (4-x)\frac{1}{2\sqrt{x+1}} + \sqrt{x+1}(-1) = 0$$

$$\frac{(4-x) - 1(2x+2)}{} = 0$$

$$2 - 3x = 0$$

$$2/3 = x$$

So $c = 2/3$