MATH 151 | **EXAM IA** | NAME Answers
Mrs. Bonny Tighe | 100 points
1.1-3.8 | SECTION_______ Mon 3/6/06

There are 11 problems at 10 points each.

1. Find dy/dx for the following:

a) $y = 2x^3 - x\sqrt{x} + \dfrac{2}{x^3} + 4$

$y = 2x^3 - x^{3/2} + 2x^{-3} + 4$

$dy/dx = 6x^2 - \frac{3}{2}x^{1/2} - 6x^{-4} + 0$

$dy/dx = 6x^2 - \frac{3}{2}\sqrt{x} - \dfrac{6}{x^4}$

b) $y = \sqrt{x^3 - 3x + 5}$

$dy/dx = \frac{1}{2}(x^3 - 3x + 5)^{-1/2}(3x^2 - 3)$

$dy/dx = \dfrac{3x^2 - 3}{2\sqrt{x^3 - 3x + 5}}$

2. Prove the following limit using the precise definition of a limit, (δ *and* ε).

$\lim_{x \to 1}(1 + x^2) = 2$

Def: $\lim_{x \to a} f(x) = L$ if for any $\varepsilon > 0$ there exists a $\delta > 0$ so that $|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$.

$|1 + x^2 - 2| < \varepsilon$ when $|x - 1| < \delta$

$|x^2 - 1| < \varepsilon$

$|x+1| \cdot |x-1| < \varepsilon$

$|x - 1| < \varepsilon / |x+1|$

let $\delta = 1$ so $|x - 1| < 1$, $-1 < x - 1 < 1$

$1 < x + 1 < 3$

$|x + 1| < 3$

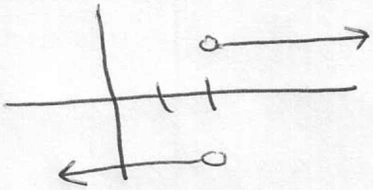
So choose $\delta = \min\{1, \varepsilon/3\}$

Check: $|x - 1| < \varepsilon/3$ so $3|x - 1| < \varepsilon$ so $|x+1| \cdot |x-1| < \varepsilon$

$|x^2 - 1| < \varepsilon$ so $|1 + x^2 - 2| < \varepsilon$ ✓

3. Find the following limits:

a) $\lim\limits_{x \to 2^-} \dfrac{x-2}{|x-2|} = $ -1



b) $\lim\limits_{x \to 3} \dfrac{\sqrt{x+1}-2}{3-x} = $ $-1/4$

$$\lim_{x\to 3} \frac{\sqrt{x+1}-2}{3-x}\left(\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}\right) = \frac{x+1-4}{(3-x)(\sqrt{x+1}+2)}$$

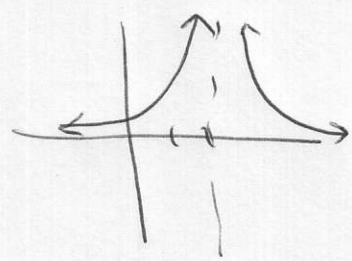
$$\lim_{x\to 3} \frac{-1\,\cancel{x-3}}{\cancel{(3-x)}(\sqrt{x+1}+2)} = \lim_{x\to 3} \frac{-1}{\sqrt{x+1}+2}$$

c) $\lim\limits_{x \to -3} \dfrac{2x^2+5x-3}{x+3} = $ -7

$$\lim_{x\to -3} \frac{(2x-1)\cancel{(x+3)}}{\cancel{(x+3)}}$$

$-6-1$

d) $\lim\limits_{x \to 2^-} \dfrac{3}{(x-2)^2} = $ $+\infty$



4. For $f(x) = \dfrac{2}{1-x}$, find the derivative of $f(x)$ using the **definition** of derivative.

Def:

$$\boxed{\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}}$$

$$\lim_{h\to 0} \left(\frac{\frac{2}{1-(x+h)} - \frac{2}{1-x}}{h}\right) =$$

$$\lim_{h\to 0} \frac{2(1-x)-2(1-(x+h))}{(1-(x+h))(1-x)h} =$$

$$\lim_{h\to 0} \frac{\cancel{2}-\cancel{2x}-\cancel{2}+\cancel{2x}+2h}{(1-(x+h))(1-x)h} = \lim_{h\to 0} \frac{2\cancel{h}}{(1-x-h)(1-x)\cancel{h}} =$$

$$\frac{2}{(1-x)(1-x)} = \frac{2}{(1-x)^2}$$

$$\lim_{h\to 0}\left(\frac{\frac{1}{2x+h}-\frac{1}{2x}}{h}\right) =$$

$$\lim_{h\to 0}\frac{2x-(2x+h)}{(2x+h)(2x)(h)}$$

5. Find the following limits :

a) $\lim_{x\to 0}\frac{\sin 6x}{6x} =$ 6

$\frac{6}{1}$

$\frac{6}{1}(1)$

b) $\lim_{h\to 0}\frac{(2x+h)^{-1}-(2x)^{-1}}{h} =$ $\frac{-1}{4x^2}$

$$\lim_{h\to 0}\frac{\cancel{2x}-\cancel{2x}-h}{(2x+h)(2x)(h)} =$$

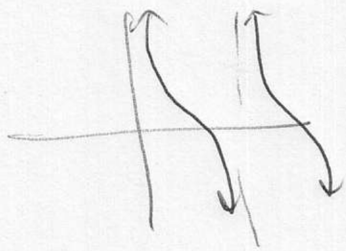
$$\lim_{h\to 0}\frac{-\cancel{h}\cdot 1}{(2x+h)(2x)(\cancel{h})} = \frac{-1}{(2x)(2x)}$$

c) $\lim_{x\to 0}\frac{\sin 3x}{\sin 4x} =$ $3/4$

$$\lim_{x\to 0}\frac{\sin 3x}{3x}\cdot\frac{4x}{\sin 4x}\left(\frac{3}{4}\right)$$

$$(1)(1)(3/4)$$

d) $\lim_{x\to\pi^+}\cot x =$ $+\infty$



6. Find an equation of the tangent line to the curve $f(x)=\cos 2x-\sin x$ at the point $(\pi/6,\ (\sqrt{3}-1)/2)$.

$f'(x)=m=-\sin 2x(2)-\cos x$ at $\pi/6$

$m=-2\sin\pi/3-\cos\pi/6$

$-2(\sqrt{3}/2)-\sqrt{3}/2 = -3\sqrt{3}/2$

$y-y_1=m(x-x_1)$

$$y-\left(\frac{\sqrt{3}-1}{2}\right)=\frac{-3\sqrt{3}}{2}\left(x-\pi/6\right)$$

7. Find the points on the curve $y=\tan^3 x$ where the tangent line is horizontal.

means $dy/dx=0$

$dy/dx = 3\tan^2 x\sec^2 x = 0$

$\tan x = 0$ $\quad \sec x \neq 0$

$\tan x = 0,\quad x = 0,\ 3\pi/2,\ -3\pi/2$

$x = 3\pi/2\, k$

8. a) Find the rate of change of M with respect to p, given $M(p)=\dfrac{2ap-p^2+l}{3a-4p}$

$$m'(p)=\frac{(3a-4p)(2a-2p)-(2ap-p^2+l)(-4)}{(3a-4p)^2}$$

b) $C(m)$ represents the cost, C, of producing m movies in dollars per minute. What does $C'(m)$ mean?

How the Cost is changing with respect to the number of movies produced in \$ per minute/movie

9. Given $h(x)=\cot 2x$ find the following:

a) $h'(\pi/6)=$ $-8/3$ b) $h''(\pi/8)=$ 16 c) $h'''(\pi/12)=$ $-896/3$

$h'(x)=-\csc^2 2x(2)$, $h'(\pi/6)=-2\csc^2(\pi/3)=-2(2/\sqrt{3})^2=-2(\frac{4}{3})$

$h''(x)=-4\csc 2x\cdot(-\csc 2x\cot 2x)(2)=8\csc^2 2x\cot 2x$

$h''(\pi/8)=8(\csc\pi/4)^2\cot\pi/4=8(\sqrt{2})^2(1)=16$

$h'''(x)=(8\csc^2 2x)(-\csc^2 2x(2))+\cot 2x(16\csc 2x(-\csc 2x\cot 2x)(2))$

$=-16\csc^4 2x-32\cot^2 2x\csc^2 2x$

$h'''(\pi/12)=-16(\csc\pi/6)^4-32(\cot\pi/6)^2(\csc\pi/6)^2$

$=-16(2)^4-32(\frac{1}{\sqrt{3}})^2(2)^2$

$=-16(16)-32(4)(\frac{1}{3})=-256-\frac{128}{3}=\frac{-896}{3}$

10. Use implicit differentiation to find dy/dx for $\sec y = \cos x - 2xy$ product

$\sec y \tan y \, dy/dx = -\sin x - [2x \, dy/dx + y(2)]$

$\sec y \tan y \, dy/dx + 2x \, dy/dx = -\sin x - 2y$

$dy/dx = \dfrac{-\sin x - 2y}{\sec y \tan y + 2x}$

11. Differentiate and simplify the following: a) $h(x) = (x^3+3)^6(3x-2)^5$

$h'(x) = (x^3+3)^6 \, 5(3x-2)^4(3) + (3x-2)^5 \cdot 6(x^3+3)^5(3x^2)$

$= 3(x^3+3)^5(3x-2)^4\left[5(x^3+3) + 6x^2(3x-2)\right]$

or $= 3(x^3+3)^5(3x-2)^4(5x^3+15+18x^3-12x^2)$

$3(x^3+3)^5(3x-2)^4(23x^3-12x^2+15)$

b) Find f'(x) given $f(x) = \dfrac{\csc x - 2x^2}{\cos 3x + 3}$

$f'(x) = \dfrac{(\cos 3x + 3)(-\csc x \cot x - 4x) - (\csc x - 2x^2)(-\sin 3x)(3)}{(\cos 3x + 3)^2}$