MATH 151 **EXAM I** NAME Answers

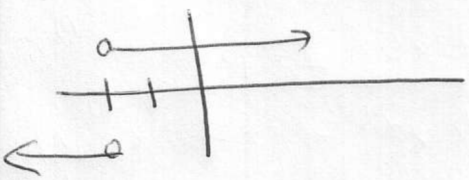
Mrs. Bonny Tighe 100 points

1.1-3.8 SECTION______ Mon 3/6/06

There are 11 problems at 10 points each.

1. Find the following limits:

a) $\lim\limits_{x\to -2^-} \dfrac{x+2}{|x+2|} =$ -1



b) $\lim\limits_{x\to 1} \dfrac{\sqrt{x+3}-2}{1-x} =$ $-1/4$

$\lim\limits_{x\to 1} \dfrac{\sqrt{x+3}-2}{1-x}\left(\dfrac{\sqrt{x+3}+2}{\sqrt{x+3}+2}\right) = \dfrac{x+3-4}{(1-x)(\sqrt{x+3}+2)}$

$\lim\limits_{x\to 1} \dfrac{-1\cancel{(x-1)}}{\cancel{(1-x)}(\sqrt{x+3}+2)} = \dfrac{-1}{\sqrt{4}+2} = -1/4$

c) $\lim\limits_{x\to -3} \dfrac{9-x^2}{x+3} =$ 6

$\lim\limits_{x\to -3} \dfrac{\cancel{(3+x)}(3-x)}{\cancel{x+3}}$

d) $\lim\limits_{x\to 2^+} \dfrac{3}{(x-2)^2} =$ $+\infty$

2. Find the following limits :

a) $\lim\limits_{x\to 0} \dfrac{\sin 3x}{x} =$ 3

$\left(\dfrac{3}{1}\right)\lim\limits_{x\to 0} \dfrac{\sin 3x}{3x} = \dfrac{3}{1}(1)$

b) $\lim\limits_{h\to 0} \dfrac{(x+h)^{-1}-x^{-1}}{h} =$ $-1/x^2$

$\lim\limits_{h\to 0} \dfrac{\frac{1}{x+h}-\frac{1}{x}}{h} = \lim\limits_{h\to 0} \dfrac{x-(x+h)}{(x+h)x\,h} =$

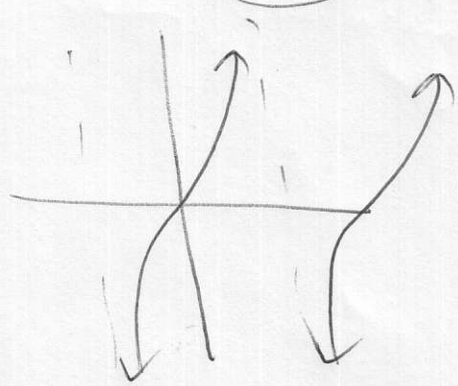
$\lim\limits_{h\to 0} \dfrac{-\cancel{h}}{(x+h)x\cancel{h}} = \dfrac{-1}{x^2}$

c) $\lim\limits_{x\to 0} \dfrac{\sin 2x}{\sin 5x} =$ $2/5$

$\lim\limits_{x\to 0} \dfrac{\sin 2x}{\cancel{2x}} \cdot \dfrac{\cancel{5x}}{\sin 5x}\left(\dfrac{2}{5}\right)$

$(1)\,(1)(2/5)$

d) $\lim\limits_{x\to \pi/2^+} \tan x =$ $-\infty$



$y - y_1 = m(x - x_1)$

3. Find an equation of the tangent line to the curve $f(x) = \sin 2x - \cos x$ at the point $(\pi/6, 0)$.

$m = f'(x) = \cos 2x (2) + \sin x$ at $x = \pi/6$

$= (\cos \pi/3)(2) + \sin \pi/6$

$1/2(2) + 1/2 = 3/2$

$y - 0 = m(x - \pi/6)$

$y = 3/2(x - \pi/6)$

4. Find the points on the curve $y = \sin^3 x$ where the tangent line is horizontal.

means $dy/dx = 0$
(slope of tangent line)

$dy/dx = 3\sin^2 x(\cos x) = 0$

$\sin x = 0 \quad \cos x = 0$

$x = 0, \pi, 2\pi \ldots$

$x = \pi/2, 3\pi/2, \ldots$

So

$x = \pi/2\, k$

5. Find dy/dx for the following:

a) $y = 3x - x^3\sqrt{x} + \dfrac{2}{x^2} + 4$

$= 3x - x^{7/2} + 2x^{-2} + 4$

$dy/dx = 3 - \frac{7}{2}x^{5/2} - 4x^{-3} + 0$

$dy/dx = 3 - \frac{7}{2}x^{5/2} - \dfrac{4}{x^3}$

b) $y = \sqrt{2x^2 + 3x + 5}$

$dy/dx = \frac{1}{2}(2x^2 + 3x + 5)^{-1/2}(4x + 3)$

$= \dfrac{4x + 3}{2\sqrt{2x^2 + 3x + 5}}$

6. Prove the following limit using the precise definition of a limit, (δ *and* ε).

$\lim_{x \to 1} (3 - x^2) = 2$

Def $\lim_{x \to a} f(x) = L$ if for any $\varepsilon > 0$ there exists a corresponding $\delta > 0$ so that $|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$.

$|3 - x^2 - 2| < \varepsilon$ when $|x-1| < \delta$

$|1 - x^2| < \varepsilon$

$|1+x|\,|1-x| < \varepsilon$

$|x-1| < \varepsilon / |1+x|$

let $\delta = 1$ so $|x-1| < \delta$ so $-1 < x-1 < 1$

$0 < x+1 < 3$

$|x+1| < 3$

so choose $\delta = \min\{1, \varepsilon/3\}$

Check: $\cancel{x}$ $|x-1| < \varepsilon/3$ so $3|x-1| < \varepsilon$ so $|x+1| \cdot |x-1| < \varepsilon$

so $|x^2 - 1| < \varepsilon$ so $|1 - x^2| < \varepsilon$ so $|3 - x^2 - 2| < \varepsilon$ ✓

7. For f(x) = $\sqrt{x-2}$, find the derivative of f(x) using the **definition** of derivative.

$$\boxed{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$$\lim_{h \to 0} \frac{\sqrt{x+h-2} - \sqrt{x-2}}{h}\left(\frac{\sqrt{x+h-2} + \sqrt{x-2}}{\sqrt{x+h-2} + \sqrt{x-2}}\right) =$$

$$\lim_{h \to 0} \frac{x+h-2 - (x-2)}{h(\sqrt{x+h-2} + \sqrt{x-2})} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h-2} + \sqrt{x-2})} =$$

$$\frac{1}{\sqrt{x-2} + \sqrt{x-2}} = \frac{1}{2\sqrt{x-2}}$$

8. Differentiate and simplify the following: a) $h(x) = (x^3+4)^4(2x-1)^5$

$h'(x) = (x^3+4)^4 \cdot 5(2x-1)^4(2) + (2x-1)^5 \cdot 4(x^3+4)^3(3x^2)$

$= 2(x^3+4)^3(2x-1)^4\left[5(x^3+4) + 6x^2(2x-1)\right]$ or

$2(x^3+4)^3(2x-1)^4\left[5x^3+20+12x^3-6x^2\right] =$

$2(x^3+4)^3(2x-1)^4(17x^3-6x^2+20)$

b) Find f'(x) given $f(x) = \dfrac{\sec x - x^2}{\tan 3x + 3}$

$f'(x) = \dfrac{(\tan 3x+3)(\sec x \tan x - 2x) - (\sec x - x^2)(\sec^2 3x\,(3))}{(\tan 3x+3)^2}$

product

9. Use implicit differentiation to find dy/dx for $xy + 3\sin y = \cos x$

$x \cdot \dfrac{dy}{dx} + y(1) + 3\cos y\, dy/dx = -\sin x$

$x\, dy/dx + 3\cos y\, dy/dx = -\sin x - y$

$dy/dx = \dfrac{-\sin x - y}{x + 3\cos y}$

10. Given $h(x) = \csc 2x$ find the following:

a) $h'(\pi/6) = \frac{-4}{3}$ b) $h''(\pi/8) = 12\sqrt{2}$ c) $h'''(\pi/12) = -208\sqrt{3}$

$h'(x) = -\csc 2x \cot 2x (2)$, $h'(\pi/6) = -2 \csc \pi/3 \cot \pi/3 = -2\left(\frac{2}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)$

$= -2\csc 2x \cot 2x$

$h''(x) = (-2\csc 2x)(-\csc^2 2x)(2) + \cot 2x(+2\csc 2x \cot 2x)(2)$

$= 4\csc^3 2x + 4\csc 2x \cot^2 2x$ at $\pi/8$ $= 4\csc^3 \pi/4 + 4\csc \pi/4 \cdot \cot^2 \pi/4$

$= 4(\sqrt{2})^3 + 4(\sqrt{2})(1)^2 =$

$8\sqrt{2} + 4\sqrt{2} = 12\sqrt{2}$

$h'''(x) = 12\csc^2 2x(-\csc 2x \cot 2x) + 4\csc 2x(2\cot 2x)(-\csc^2 2x) + \cot^2 2x(4\csc 2x \cot 2x)$

$= -12\csc^3 2x \cot 2x - 8\csc^3 2x \cot 2x - 8\cot^3 2x \csc 2x$

$= -20\csc^3 2x \cot 2x - 8\cot^3 2x \csc 2x$ at $\pi/12$

$-20\csc^3 \pi/6 \cot \pi/6 - 8\cot^3 \pi/6 \csc \pi/6 = -20(2)^3(\sqrt{3}) - 8(\sqrt{3})^3(2)$

11. a) ~~Find the rate of change of M with respect to p, given~~ $M(p) = \frac{2ap - p^3}{3a + p}$

$$M'(p) = \frac{(3a+p)(2a-3p^2) - (2ap - p^3)(1)}{(3a+p)^2}$$

b) $C(m)$ represents the cost, C, of producing m movies in dollars per minute. What does $C'(m)$ mean?

The rate of change of cost with respect to the number of movies produced in ~~movies~~ \$ per minute / movie

$-20(8)\sqrt{3} - 8(3\sqrt{3})2$

$-160\sqrt{3} - 48\sqrt{3}$