Applying the Optimal Interpolation Data Assimilation Method to an S-E-I-R-D Model to a Simulated Ebola Epidemic and to Forecast the Coronavirus (COVID-19) Pandemic in Nigeria

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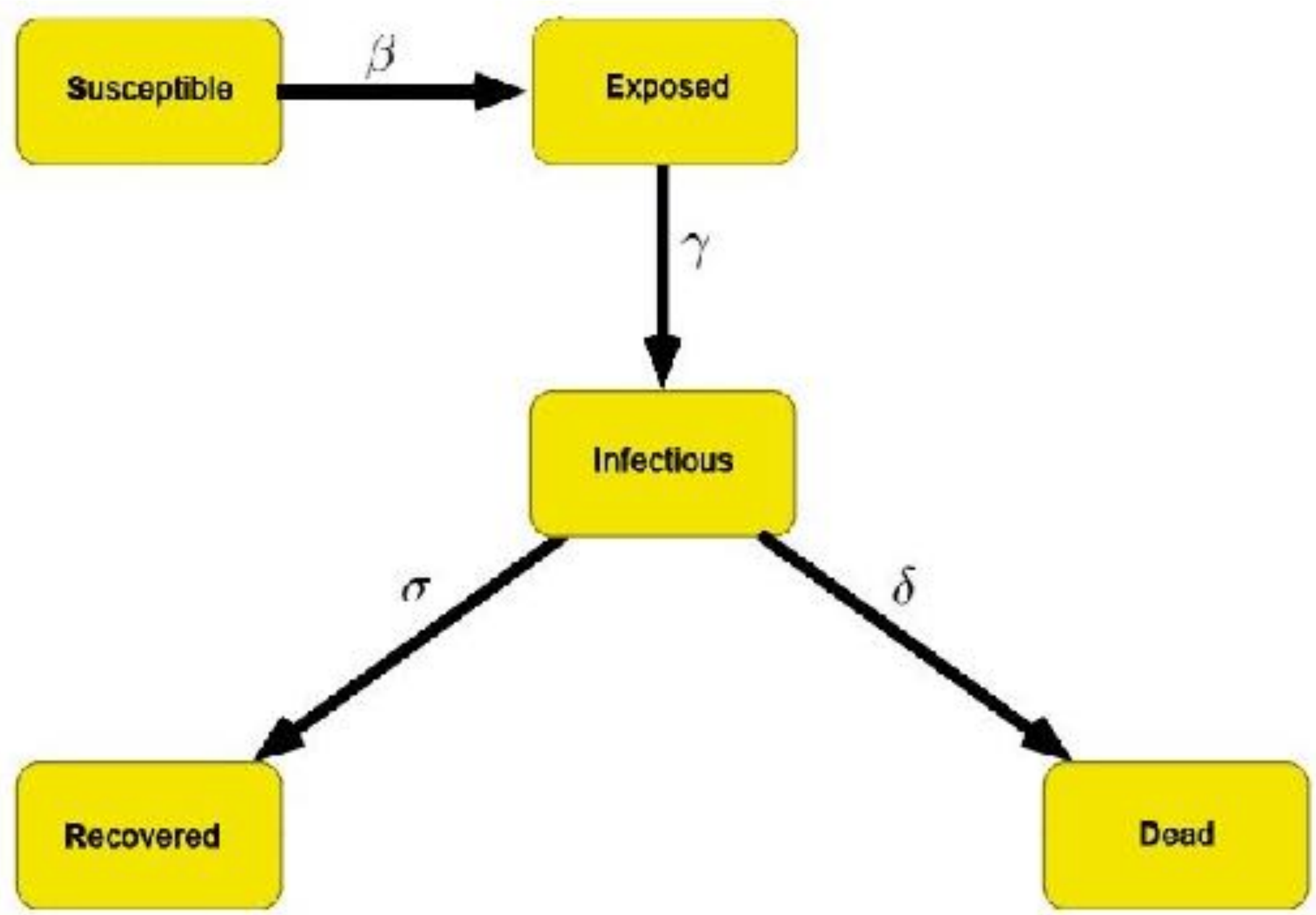
Overview:

1. The Compartmental Model of an Infectious Disease: SEIRD
2. Running the SEIRD Model Simulation: Nigeria
3. Optimal Interpolation Data Assimilation
4. Insights and Challenges of Forecasting COVID-19 in Nigeria

The Compartmental Epidemic Model of an Infectious Disease: SEIRD

S	**Susceptible**	number of subjects who are susceptible but not yet infected; base pool of persons
E	**Exposed**	number of subjects who are infected but not yet infectious; in an incubatory stage where they cannot yet transmit the disease
I	**Infectious**	number that are infected and can transmit the disease
R	**Recovered**	number that have received immunization, are fully recovered, or quarantined; cannot transmit the disease
D	**Dead**	number confirmed to have died from the disease

The Compartmental Epidemic Model of an Infectious Disease: SEIRD



Parameter	Description
β	Daily fraction that move out of the susceptible compartment into the exposed compartment
γ	Daily fraction that move out of the exposed compartment into the infectious compartment
σ	Daily fraction that move out of the infectious compartment into the recovered compartment
δ	Daily fraction that move out of the recovered compartment into the dead compartment

The Compartmental Epidemic Model of an Infectious Disease: SEIRD

SEIRD Model: Epidemic Dynamics in Continuous Time:
a system of five PDEs to describe spatio-temporal evolution over connected planar domain $\Omega \subset R^2$

$$\frac{\partial S(x,y,t)}{\partial t} = -\beta S(x,y,t)\iint_{\Omega} \underline{w(x,y,u,v)}\, I(u,v,t)\,dudv$$

weight function

$$\frac{\partial E(x,y,t)}{\partial t} = \beta S(x,y,t)\iint_{\Omega} w(x,y,u,v)\, I(u,v,t)\,dudv - \gamma \underline{E(x,y,t)}$$

$$\frac{\partial I(x,y,t)}{\partial t} = \gamma E(x,y,t) - \sigma I(x,y,t) - \delta I(x,y,t)$$

The function for each SEIRD variable describes the density of the population for each compartment at spatial coordinate (x,y) and time *t*.
For example, *E(x,y,t)* describes the density of the exposed population at spatial point (x,y) and time *t.*

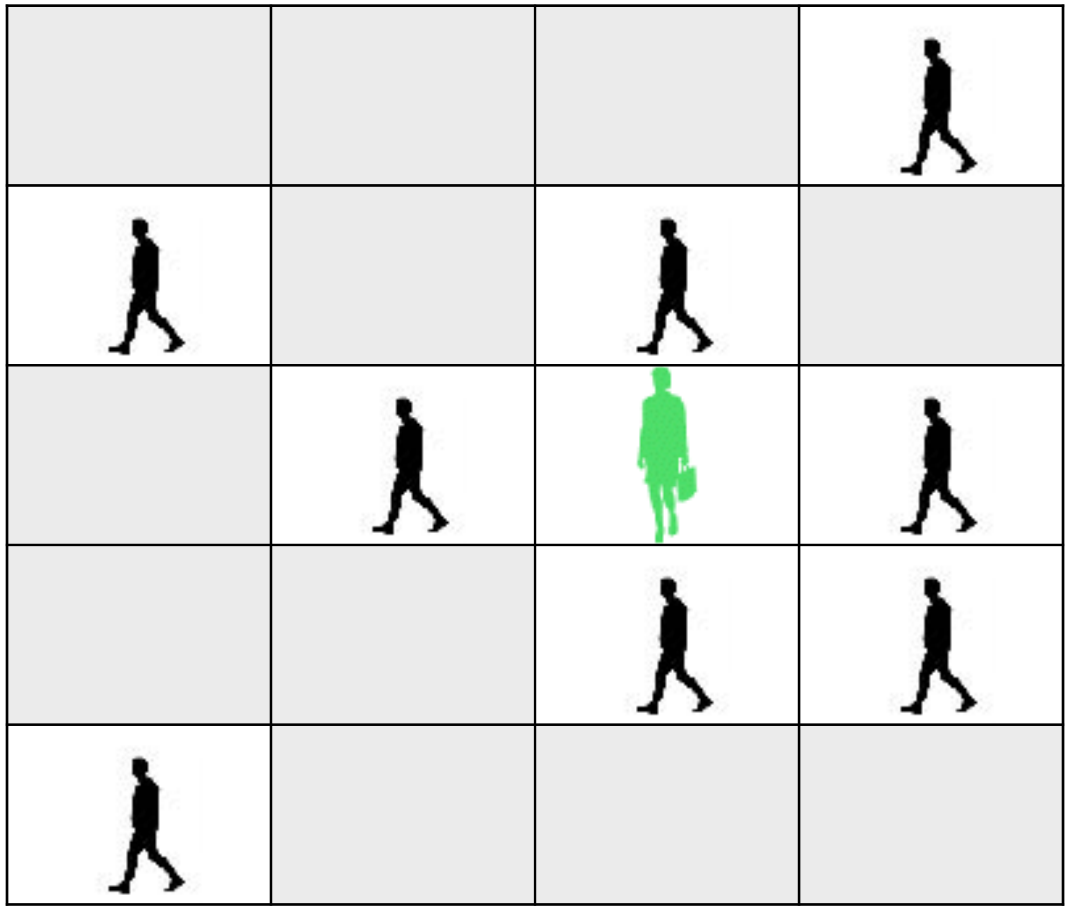
$$\frac{\partial R(x,y,t)}{\partial t} = \sigma I(x,y,t)$$

$$\frac{\partial D(x,y,t)}{\partial t} = \delta I(x,y,t)$$

The Compartmental Epidemic Model of an Infectious Disease: SEIRD

Weight Function:

$$w(x,y,u,v) \propto \exp\left(-\sqrt{(x-u)^2+(y-v)^2}\,/\lambda\right)$$



- weight function measures influence of infectives at (u,v) on exposure of susceptibles at (x,y)
- expresses idea that influence of nearby infectives drops as an exponential function of Euclidean distance
- The more mobile the society, the higher the λ value (constant characteristic of the distance the disease spreads)
- This distance parameter is adequate for capturing local dynamics, thereby allowing us to learn about spatial transmission of the disease across neighboring cells

The Compartmental Epidemic Model of an Infectious Disease: SEIRD

We simulate these epidemic dynamics by utilizing a discretized stochastic version of the model with the assumption individuals are continuously distributed on a spatial domain. The five PDEs are described as a system of five ODEs:

$$\frac{dS}{dt} = -\beta\frac{S(t)I(t)}{N(t)}$$

$$\frac{dE}{dt} = \beta\frac{S(t)I(t)}{N(t)} - \gamma E(t)$$

$$\frac{dI}{dt} = \gamma E(t) - \sigma I(t) - \delta I(t)$$

$$\frac{dR}{dt} = \sigma I(t)$$

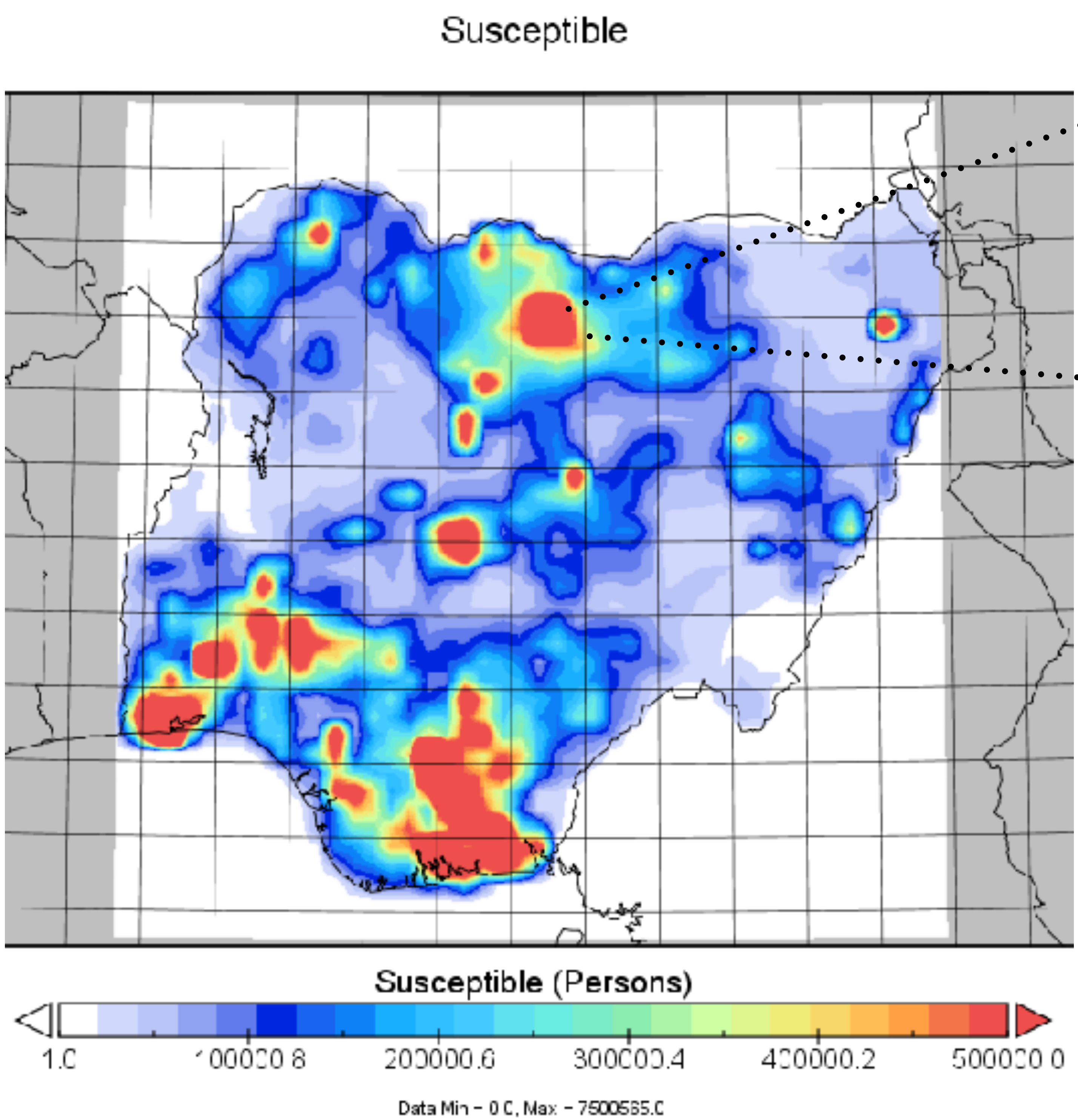
$$\frac{dD}{dt} = \delta I(t)$$

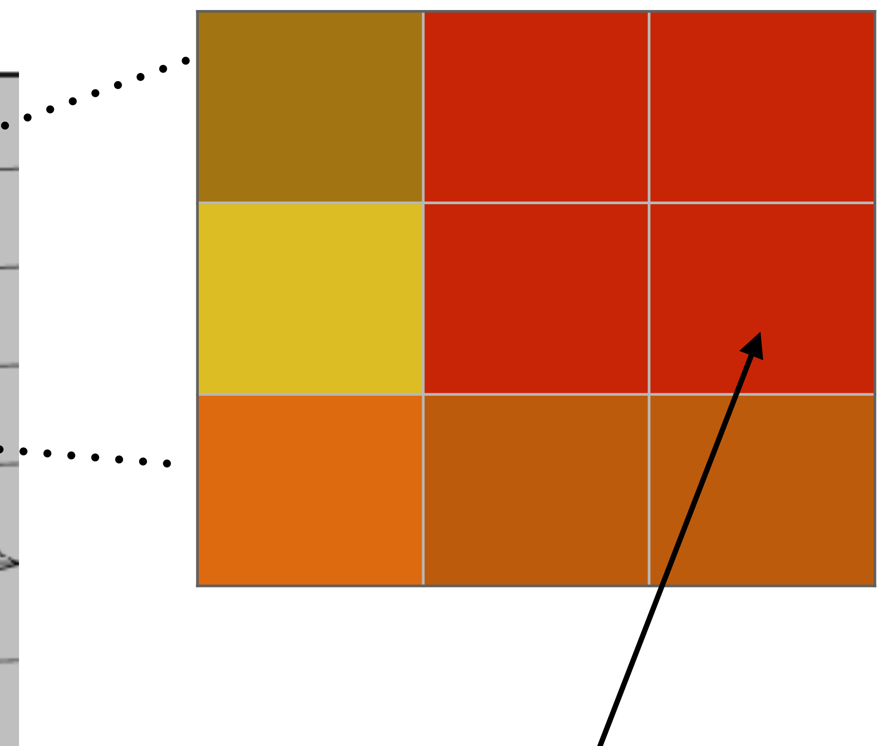
$$N(t) = S(t) + E(t) + I(t) + R(t)$$

Running the SEIRD Model Simulation: Nigeria

- Synthetic disease incidence data for three cities in Nigeria: Abuja and Gombe
- Artificial simulation of an Ebola outbreak
- Assume data has been collected every Sunday once a week for 2017, 2018, and 2019
- 1 time step = 1 day ; 1,095 time steps total for three years
- Every month, a visual image of the epidemic spread for each compartment is outputted by the R program as a netCDF file and visualized in Panoply

Running the SEIRD Model Simulation: Nigeria





(high density of susceptibles in this particular pixel)

Each cell size (~2.5 km, ~2.5 km) at equator for a total of 3480 rows x 8640 columns = 30067200 non-overlapping cells at 2.5 arc-minute resolution.

Running the SEIRD Model Simulation: Nigeria

```
        #--------------------------------#
        # Store the next state of the cell #
        #--------------------------------#

        nextSusceptible[i,j] <- S[i,j] - newVaccinated - newExposed
  #      nextVaccinated[i,j] <- V[i,j] + newVaccinated
        nextExposed[i,j] <- E[i,j] + newExposed - newInfected
        nextInfected[i,j] <- I[i,j] + newInfected - newDead - newRecovered
        nextRecovered[i,j] <- R[i,j] + newRecovered
        nextDead[i,j] <- D[i,j] + newDead
      }               # nLiving
    }                 # Inhabitable
  }                     # ncols
}                       # nrows

nextSusceptible[nextSusceptible < 0] = 0
# nextVaccinated[nextVaccinated < 0] = 0
nextExposed[nextExposed < 0] = 0
nextInfected[nextInfected < 0] = 0
nextRecovered[nextRecovered < 0] = 0
nextDead[nextDead < 0] = 0

S <- nextSusceptible
#  V <- nextVaccinated
E <- nextExposed
I <- nextInfected
R <- nextRecovered
D <- nextDead

summary[t, 8]   <- cumExposed
summary[t, 9]  <- dailyIncidence
summary[t, 10]  <- cumIncidence

########## DA Begins ##########
```

Optimal Interpolation Data Assimilation

Finds the optimal estimate x^a of the true state of the system given the background field x^b, incoming observations y^0, and the error covariance matrices of the background $B \in \mathbb{R}^{n \times n}$ and observations $R \in \mathbb{R}^{p \times p}$

The weight matrix W minimizes mean-square error $E\{\epsilon^T \epsilon\}$.
The observational operator H transforms modeled variable $x(t)$ such that it can be compared to the observation $y(t)$.

$$\mathbf{x}^a = \mathbf{x}^b + \mathbf{W}\left[\mathbf{y}^o - H(\mathbf{x}^b)\right] = \mathbf{x}^b + \mathbf{W}\mathbf{d},$$
$$\mathbf{W} = \mathbf{B}\mathbf{H}^T(\mathbf{R} + \mathbf{H}\mathbf{B}\mathbf{H}^T)^{-1},$$
$$\mathbf{P}^a = (\mathbf{I} - \mathbf{W}\mathbf{H})\mathbf{B}.$$

Optimal Interpolation Data Assimilation

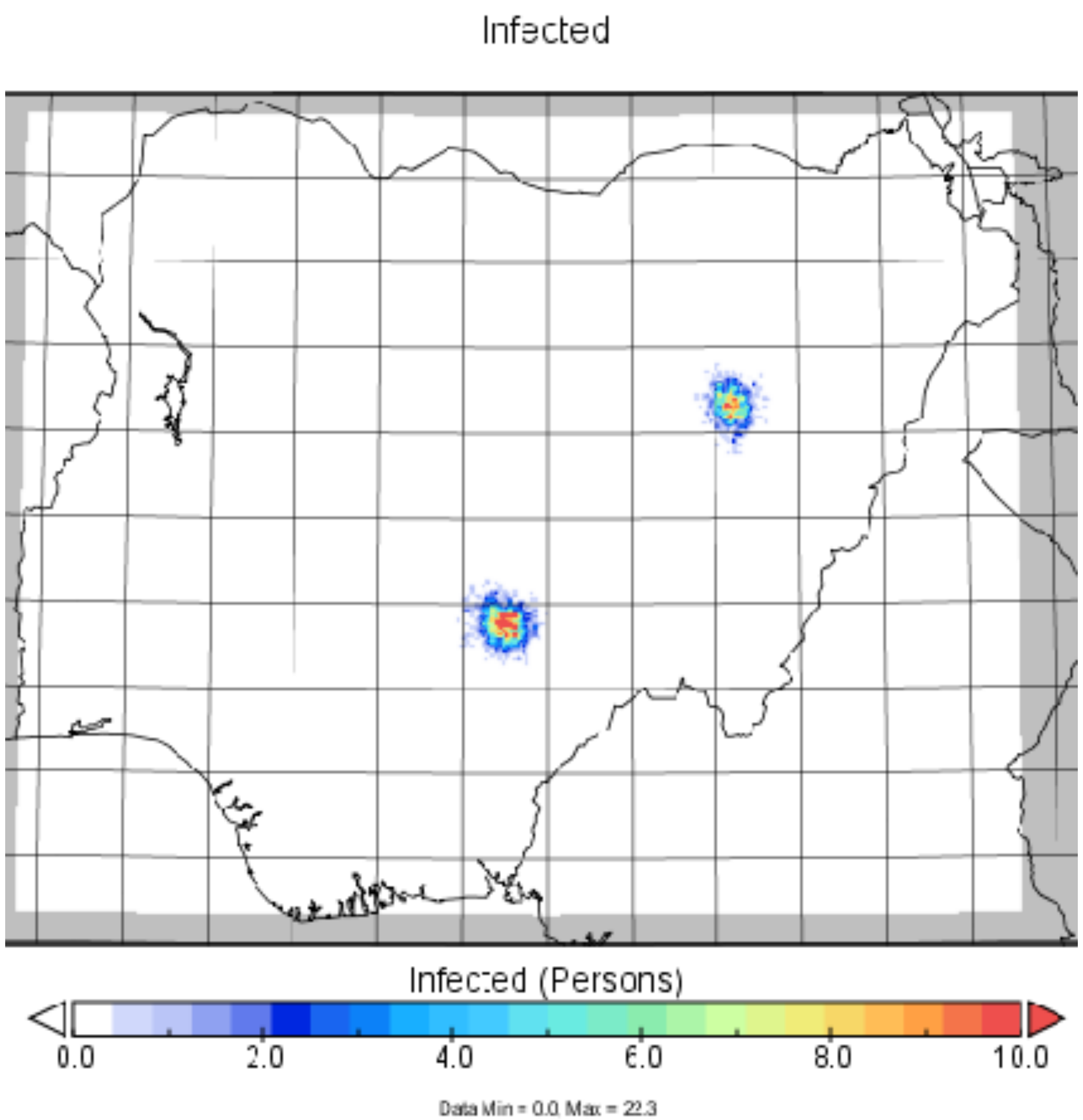
Random vectors $x(t)$, the analysis of the true state, and $y(t)$, the observations, are represented as parallel time-series for each spatial location.

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}, \qquad \mathbf{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_p(t) \end{bmatrix},$$

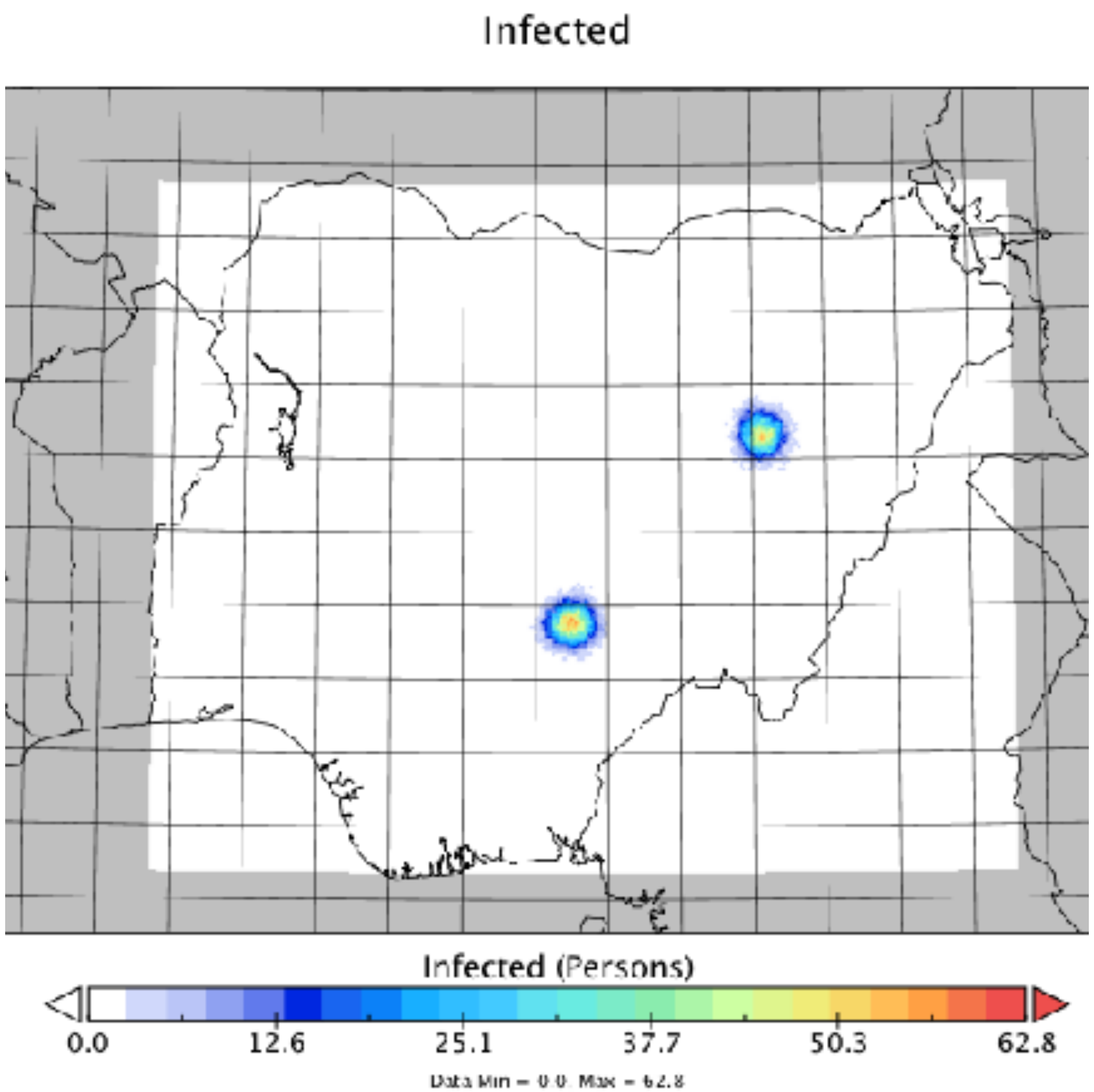
OI assumes (a) inherent variability in scalar field of interest, and that (b) the observations are error-free.

Optimal Interpolation Data Assimilation

t = 80 days



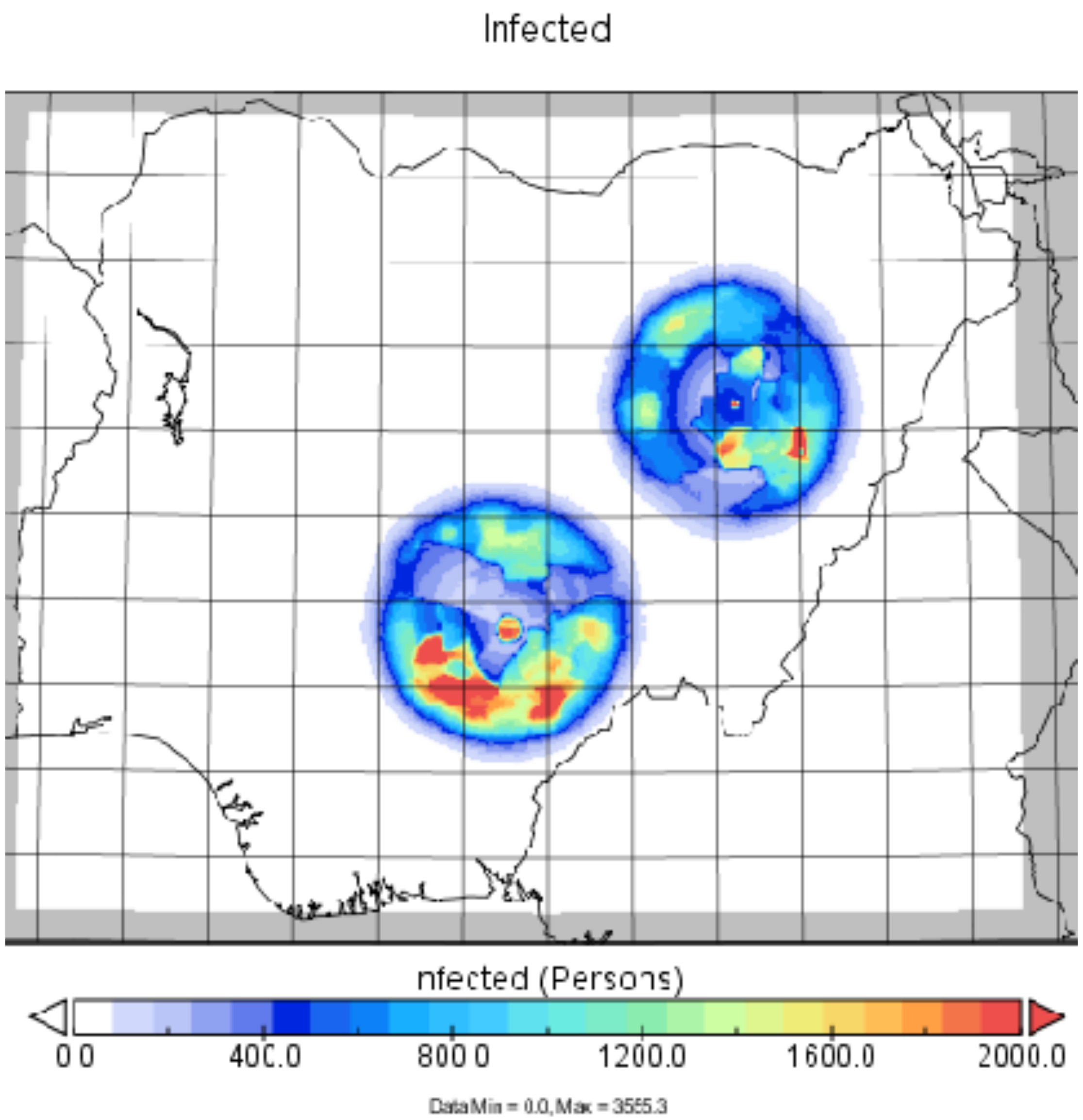
Simulation of SEIRD epidemic



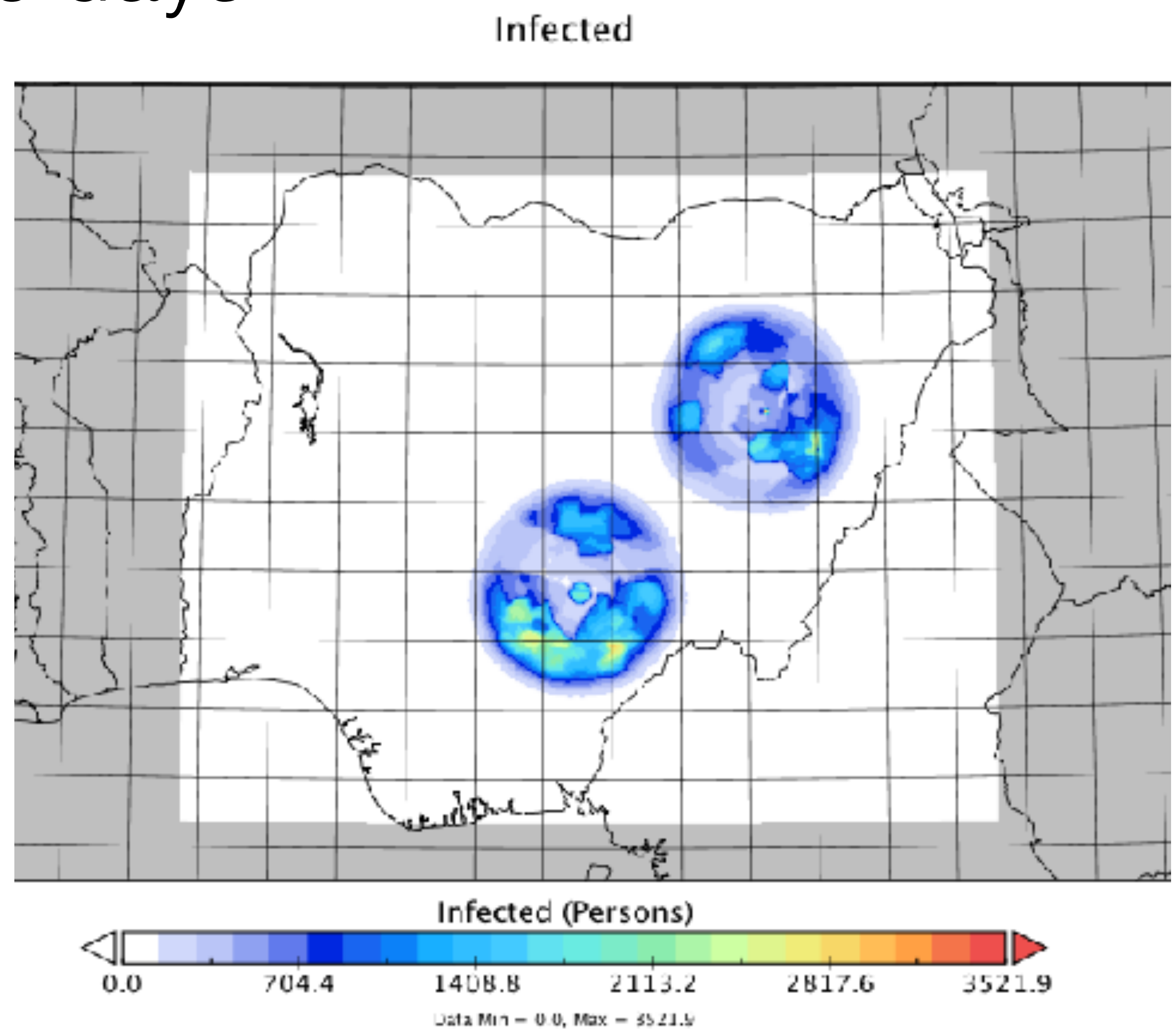
Optimal Interpolation Forecast

Optimal Interpolation Data Assimilation

t = 250 days



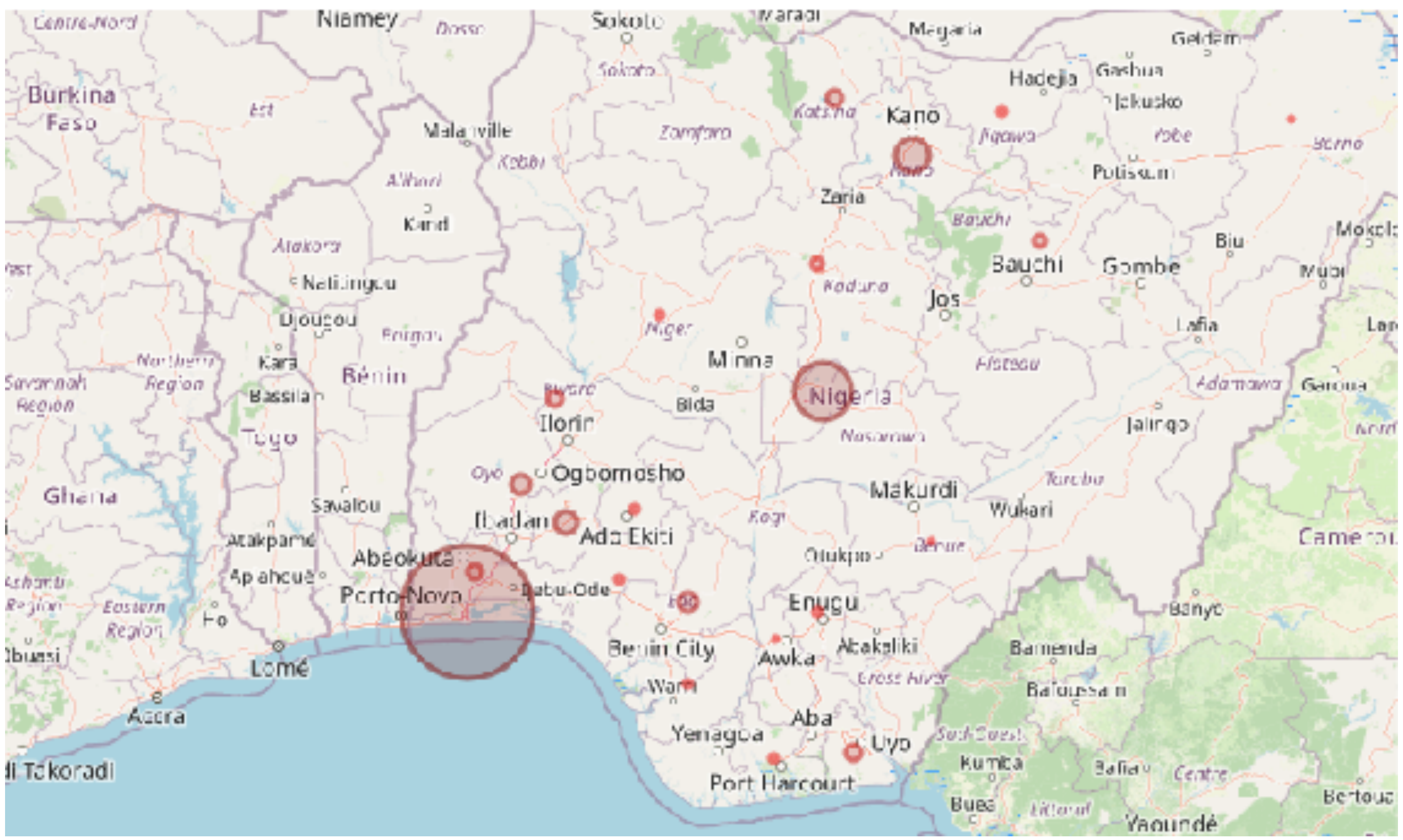
Simulation of SEIRD epidemic



Optimal Interpolation Forecast

The first case of COVID-19 in Nigeria was confirmed for Lagos State on February 27, 2020.

As of April 19, 2020, there are 627 confirmed cases in 22 states, with the highest count being in Lagos.



COVID-19 cases in Nigeria (v·t·e)

Deaths | Recoveries | Active cases

Date	# of cases	
2020-02-27	1 (+1)	(n.a.)
⋮	1 (=)	
2020-03-09	2 (+1)	(+100%)
⋮	2 (=)	
2020-03-13	1	
⋮	1 (=)	
2020-03-17	2 (+1)	(+100%)
2020-03-18	7 (+5)	(+250%)
2020-03-19	11 (+4)	(+57%)
2020-03-20	12 (+1)	(+9%)
2020-03-21	22 (+10)	(+91%)
2020-03-22	30 (+8)	(+36%)
2020-03-23	40 (+10)	(+33%)
2020-03-24	44 (+4)	(+10%)
2020-03-25	51 (+7)	(+16%)
2020-03-26	65 (+14)	(+27%)
2020-03-27	81 (+16)	(+25%)
2020-03-28	97 (+16)	(+20%)
2020-03-29	111 (+14)	(+14%)
2020-03-30	131 (+20)	(+18%)
2020-03-31	151 (+20)	(+15%)
2020-04-01	174 (+23)	(+15%)
2020-04-02	184 (+10)	(+6%)
2020-04-03	209 (+25)	(+14%)
2020-04-04	214 (+5)	(+2%)
2020-04-05	232 (+18)	(+8%)
2020-04-06	238 (+6)	(+3%)
2020-04-07	254 (+16)	(+7%)
2020-04-08	274 (+20)	(+8%)
2020-04-09	288 (+14)	(+5%)
2020-04-10	305 (+17)	(+6%)
2020-04-11	318 (+13)	(+4%)
2020-04-12	323 (+5)	(+1.6%)
2020-04-13	343 (+20)	(+6.2%)

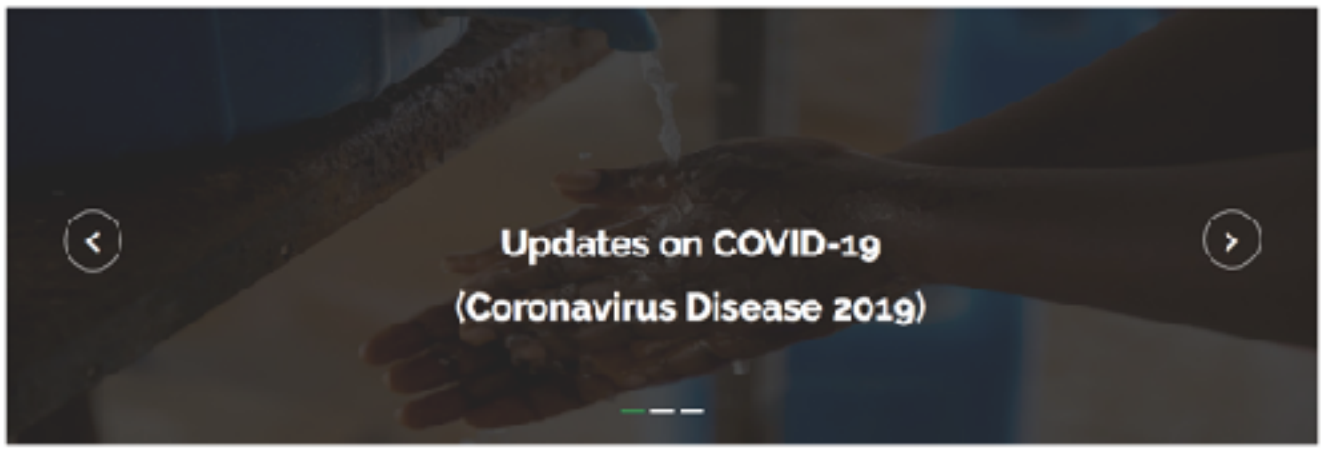
Sources: various news sources and state health department websites. See Timeline Table and Timeline narrative for sources.

Insights and Challenges of Forecasting COVID-19 in Nigeria

NCDC

Nigeria Centre for Disease Control
Protecting the health of Nigerians

Home | FAQs | Advisory | Guidelines | Resources | Situation Report | Dashboard | Main Website | Emergency Contacts



CASE SUMMARY IN NIGERIA AS AT APRIL 19TH 2020

Total Samples Tested	> 7153
Total Confirmed cases	627
Discharged	170
Death	21

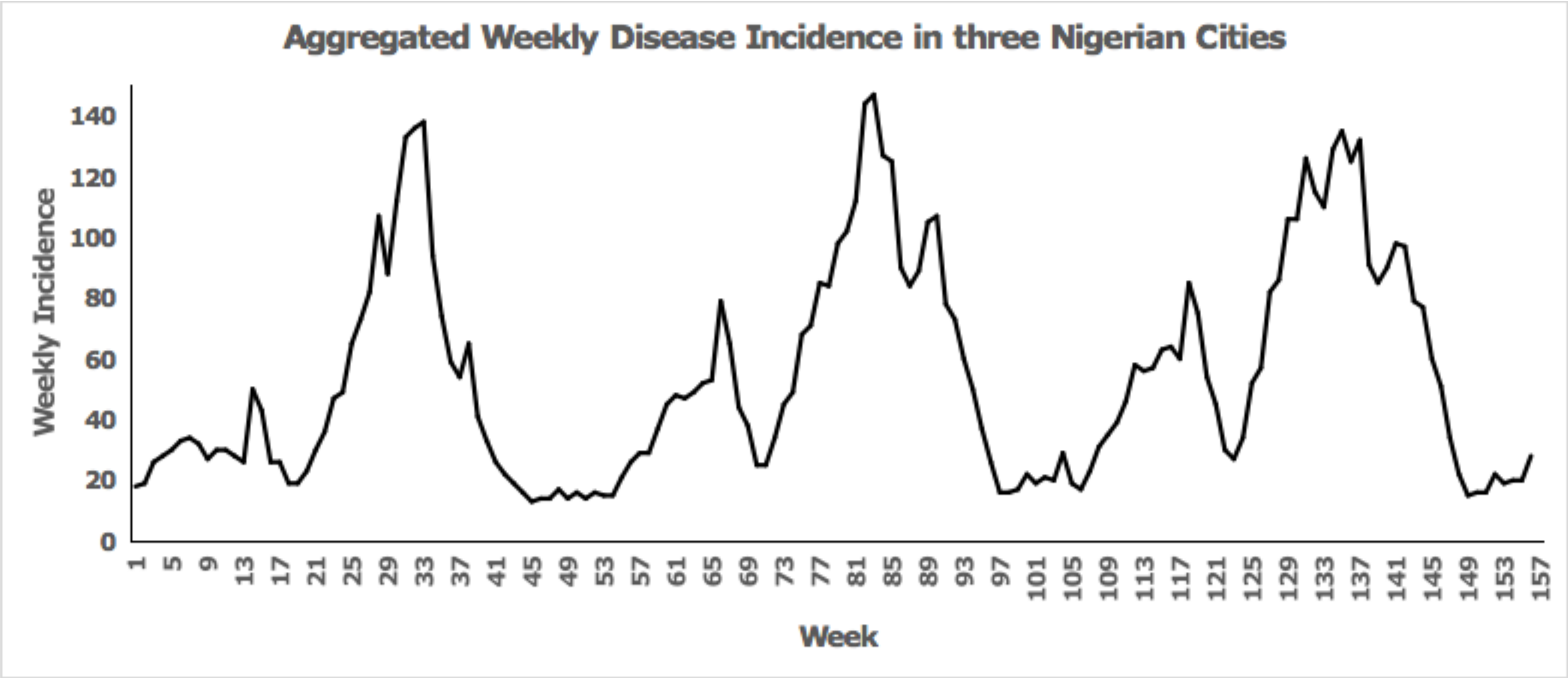
States Affected	No. of Cases (Lab Confirmed)	No. of Active Cases	No. Discharged	No of Deaths

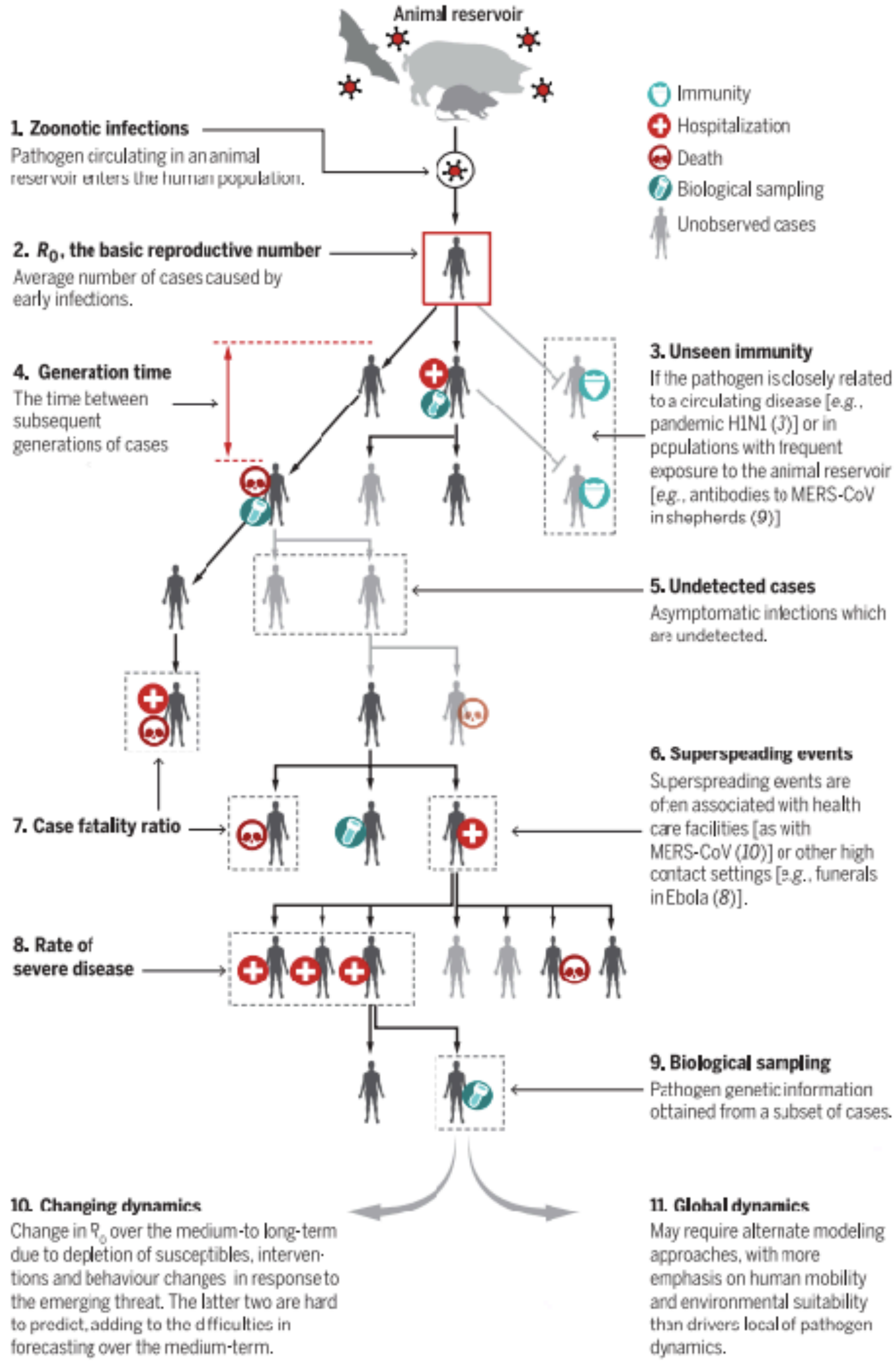
	A	AJ	AK	AL	AM	AN	AO	AP
1	States	Mar 28	Mar 29	Mar 30	Mar 31	Apr 1	Apr 2	Apr 3
14	Edo	2	2	2	2	4	4	7
15	Ekiti	1	1	1	1	2	2	2
16	Enugu	2	2	2	2	2	2	2
17	Gombe	0	0	0	0	0	0	0
18	Imo	0	0	0	0	0	0	0
19	Jigawa	0	0	0	0	0	0	0
20	Kaduna	1	1	3	3	4	4	4
21	Kano	0	0	0	0	0	0	0
22	Katsina	0	0	0	0	0	0	0
23	Kebbi	0	0	0	0	0	0	0
24	Kogi	0	0	0	0	0	0	0
25	Kwara	0	0	0	0	0	0	0
26	Lagos	59	68	81	82	91	98	109
27	Nasarawa	0	0	0	0	0	0	0
28	Niger	0	0	0	0	0	0	0
29	Ogun	3	3	3	4	4	4	4
30	Ondo	0	0	0	0	0	0	1
31	Osun	2	2	2	5	14	14	20
32	Oyo	7	7	8	8	8	8	8
33	Plateau	0	0	0	0	0	0	0
34	Rivers	1	1	1	1	1	1	1
35	Sokoto	0	0	0	0	0	0	0
36	Taraba	0	0	0	0	0	0	0
37	Yobe	0	0	0	0	0	0	0
38	Zamfara	0	0	0	0	0	0	0

CumulativeCases | ActiveCases | Recovered

Insights and Challenges of Forecasting COVID-19 in Nigeria

Estimating beta transmission rate with respect to movement from susceptible to exposed compartment in SEIRD model





(Metcalf and Lesser, 2017)

Thank you!