Numerical Simulation of Mechanical Structures

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Outline

- Motivation
- Objective
- Equation of the model
- Example of a Mechanical Structures
- Description of the Mechanical Structure
- Derivation the Mechanical Structure
- Stochastic Vibrations of the Structure
- Conclusion and Future Research

Motivation

The ultimate goal of this research is to mitigate the effects of earthquakes on structures. By deriving the mass and stiffness of the building, further research can be done to derive the damping force using the known variables. This damping force is then applied to the structure to cause little to no vibrations during wind and earthquakes.

Research Objective

1. Implementation of finite element method to simulate vibrations of a mechanical structure. Specifically, we use a 2D frame model and corresponding stiffness, mass and damping matrices to set up a system of ordinary differential equations, which is solved in MATLAB.
2. Consider uncertainties in the model parameters by taking the Young's modulus as a uniformly distributed random variable. We use Monte Carlo simulation and study the effect of uncertainties by numerical experiments.

Equation of the model

From Newton's second law of motion

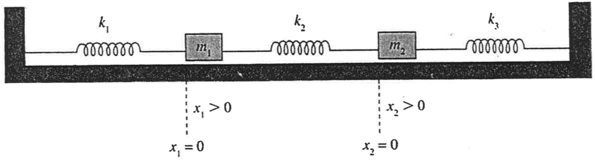
$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(t), \qquad (1)$$

which is a linear, second-order, nonhomogeneous, differential equation (resp. a system of equations) with constant coefficients, where

$x(t)$ ... displacement vector (of size n_d),
M ... mass matrix,
C ... damping matrix,
K ... stiffness matrix,
$f(t)$ ... vector of external forces.

Examples: Coupled Springs

This is the prototype for mechanical vibrations, which we studied first [4]. Specifically, we looked at



We solve the initial-value problem

$$m_1 \ddot{x}_1 = -k_1 x_1 + k_2 (x_2 - x_1),$$
$$m_2 \ddot{x}_2 = -k_2 (x_2 - x_1) - k_3 x_2,$$

with initial conditions

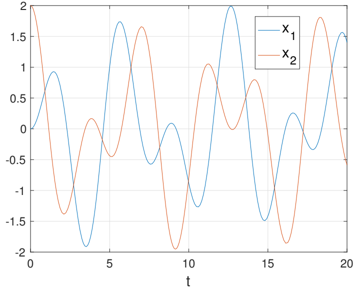
$$x_1(0) = 0, \;\; \dot{x}_1(0) = 0, \quad x_2(0) = d, \;\; \dot{x}_2(0) = 0.$$

Coupled Springs contd.

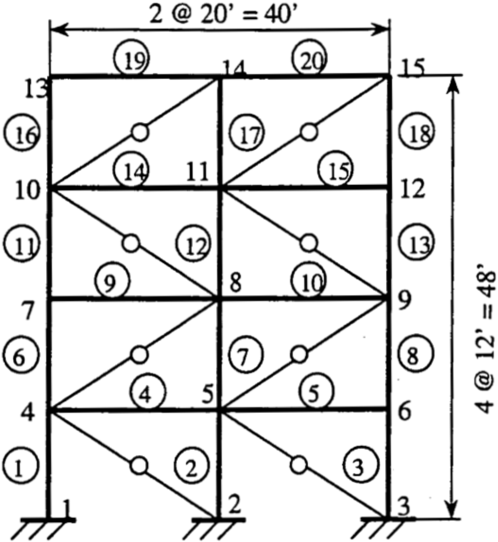
We first derived the equations and then simulated these equations into matlab using ODE solvers such as *ode*45 or *ode*23*s*(for stiff problems) to obtain the plot of the solution below. Thus we get:

$$M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}, \quad K = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix},$$

$$z(0) = \begin{bmatrix} 0 & d & 0 & 0 \end{bmatrix}^T, \quad f(t) = 0.$$



The Mechanical Structure



Description of the Mechanical Structure

We also looked at a planar structure made of frame elements, which was used in [1] as a model of a four-story building. The structure is made of 20 elements, each element has 2 nodes, and there are 3 degrees of freedom (dof) per node. We used the standard finite element model and assembled the global stiffness, damping and mass matrices. In total there are 45 degrees of freedom. Since the material of the structure is assumed to be a linear viscoelastic solid, the damping matrix has the same form as the stiffness matrix with the Young's modulus being replaced by the damping constant.

Description of the Mechanical Structure, contd.

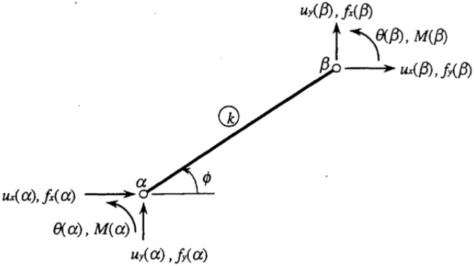


Figure: Frame Degrees of freedom

For simplicity, Young's modulus $E = 200$ psi and other parameters (density, cross-sectional area, damping constant, ...) are set to 1. The initial condition is zero, and forcing $f(t)$ is a scaled sin-wave.

Derivation of the Mechanical Structure

The equation of motion (1) is transformed into ($2n_d$-dimensional) state space representation

$$\dot{z}(t) = Az + F(t), \tag{2}$$

where

$$z(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix},$$

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix},$$

$$F(t) = \begin{bmatrix} 0 \\ M^{-1}f(t) \end{bmatrix},$$

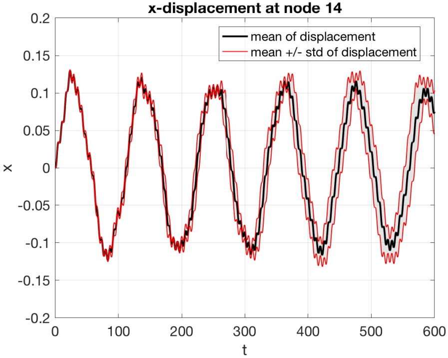
with initial conditions, for example, $z(0) = 0$.

Stochastic Vibrations of the Mechanical Structure

Monte Carlo Simulation

- We implemented the model in MATLAB and used `ode45` solver. Due to adaptive time-stepping, for Monte Carlo simulation we interpolated the results in post-processing to constant time intervals.
- We considered 10% variability of the Young's modulus E, and used Monte Carlo simulation with 10^4 samples. Specifically, we randomly sampled E from a uniform distribution in the range $190 - 210$ psi, and we simulated the motion of the planar structure in the time interval $[0, 600]$ s.

Monte Carlo plot



The horizontal displacement of the node 14 (center of the roof) is shown above. The mean displacement is given by the periodic forcing, and we see that the width of the band given by standard deviation of the displacement increases with time.

Conclusion

- We learned the basics of finite elements and MATLAB programming.
- Based on our knowledge of elementary differential equations and numerical analysis, we derived and implemented models of vibrations for several mechanical structures.
- Finally, we also applied our codes in Monte Carlo simulation.

Future work

The future research will focus on

- The implementation of active structural control,
- The use of realistic earthquake data for forcing,
- Testing the design, reliability and efficiency of the model, and its uncertainty.

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