What this course is about:

We are primarily concerned to answer the question:

Q1 How did modern mathematics come to be what it is now?

Implicit in this is some understanding of what characterizes the body of skills and knowledge we call Mathematics and the recognition that this has changed in time. There are then two closely related questions:

Q2 How do we know any of this? How much is speculation or interpretation?

Q3 Why did historical mathematicians choose to work on what they did?

Q4 What is the role of definitions in mathematics and how did the “mathematical community” come to use the ones they used?

We then ask about the roles of

examples, puzzles, analogy, applications, communication, textbooks

in determining this. Implicit in this are the questions of how mathematics is transmitted and taught and how it is valued and supported.

Q5 Can we “put ourselves in the heads” of earlier mathematicians?

Note that in doing so we will come to understand more clearly the deeper meaning of things we have become used to accepting without much conscious thought. As a final general question:

Q6 Is the development of Mathematics a general (inevitable) progress by many mathematicians or is it primarily the work of some few Great Mathematicians?
Beginning in antiquity we have the development of **counting** (and recording numbers) and **measurement**, along with the related arithmetic. We will study these in the contexts of ancient Mesopotamia and ancient Egypt and then continue with the further development of Greek mathematics and the idea of “proofs”. Our primary emphasis will be on what we understand of answers to the general questions (Q1–6) we have raised. We have learned some new facts and raised some new questions over the intervening millennia, but this ancient development is the essential background, setting the stage for our modern Mathematics. While the mathematical facts involved in this are largely quite familiar from our early grades (and we will learn the somewhat different algorithms used), there are a number of surprises as to real difficulties they worried about which we tend to ignore. Much of this comes from difficulties in the treatment of “infinity”. Nevertheless, Euclid’s approach was considered the “gold standard” of reasoning for about two millennia.

The next development we will discuss in some detail is the development of Algebra (solving polynomial equations), for which we consider the contrast between algebraic (largely algorithmic) approaches and geometric reasoning and the introduction of a “number” 0 as well as negatives and the complexes. [We will also learn how to solve a cubic!]

The third major development will be Analysis: What was new that Newton and Leibniz worked out that is often called “inventing the Calculus?” Why was this (initially) so controversial and unsatisfactory? This led to deep investigations of “What is a number?” and “What is a function?” and we may consider whether we are satisfied with the nature of the (still somewhat controversial) resolution attained.

What additional content we consider beyond this will depend on students’ backgrounds and interests as well as on the time available. I would like to try to give some idea of what Mathematics is today — I have lived through much of the change in the second half of the 20th Century — but will probably not be able to make all of this comprehensible.

Note that MATH 432 is designated a Writing Intensive Course and we will necessarily pay attention to that, but will try not to waste much class time with formal explanation of grammatical or stylistic considerations, however important — I am hoping that it will be possible to explain the writing concerns in ways which also relate to the mathematical content: for example, one might think of a mathematical proof as a special type of “Persuasive Essay” with related notions of coherence and organization.