Validation of a Model of a Resonant Optothermoacoustic Trace Gas Sensor

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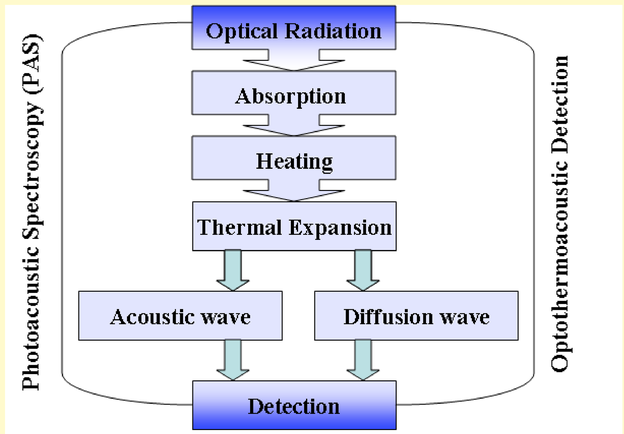
Resonant OptoThermoAcoustic DEtection (ROTADE)

ROTADE uses a quartz tuning fork (QTF) to detect diffusion waves generated when optical radiation interacts with a gas



Detection

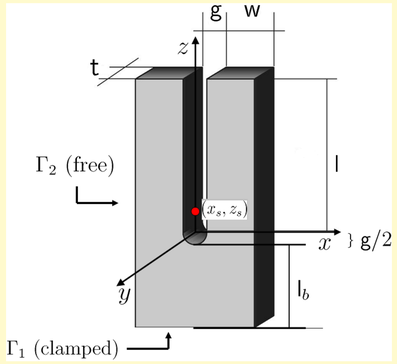
- Gas molecules diffuse and collide with QTF surface
- Excited molecules transfer vibrational energy to QTF
- Heating induces a stress and displacement of the QTF
- A resonance is induced by modulating the laser
- A current is generated via piezoelectricity of quartz

Current is proportional to gas concentration.

ROTADE Characteristics

- Effective at low ambient pressures ($\lesssim 50$ Torr)
- High wavelength selectivity
- Allows for analysis of small gas samples (< 1 mm^3)
- Offers immunity to environmental acoustic noise

Geometry of QTF



Laser beam axis is parallel to y-axis through $(0, 0, z)$

Helmholtz Equation for Diffusion Wave

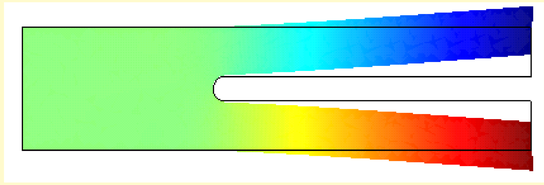
$$-\nabla \cdot (k \nabla T) + i\omega_L T = S$$

T = temperature
k = thermal diffusivity
ω_L = laser modulation frequency
S = laser source amplitude

Helmholtz Equation for Thermoelastic Deformation

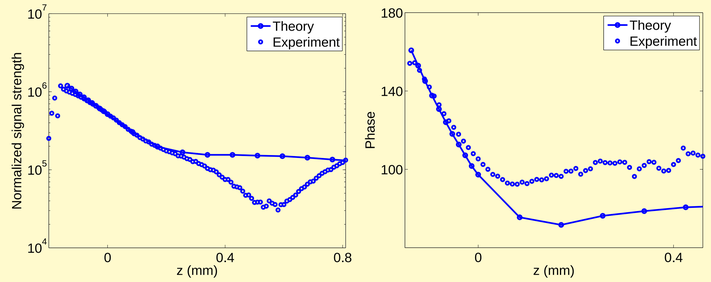
$$\nabla \cdot C[\nabla \mathbf{u}] + (\rho \omega_{\mathrm{TF}}^2 - ib\omega_{\mathrm{TF}})\,\mathbf{u} = \nabla \cdot C[\alpha T]$$

$\mathbf{u}$ = displacement field
ρ = density
b = damping constant
C = elasticity tensor
ω_{TF} = QTF resonance freq. = ω_L
α = thermal expansion tensor



32.8 kHz eigenmode of vibration of QTF

Experimental Validation

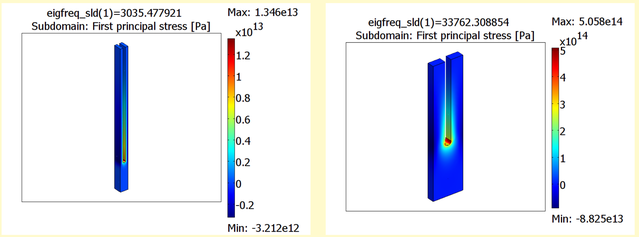


Amplitude (left) and phase (right) of signal as function of vertical position, z, of source for 0.5% C_2H_2 in N_2 at 20 Torr

- Discrepancy for $z > 0.25$ mm is due to QEPAS signal
- Signal is largest when laser is near base of QTF
- Molecular dynamics of C_2H_2 do not influence signal

Design Optimization of QTF

Numerical optimization was performed using the mesh-adaptive direct search (MADS) algorithm



First fundamental stress of a 3 kHz QTF (left) whose response is 24× that of standard 32.8 kHz QTF (right)

Further Information

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