Heat Conduction Within Linear Thermoelasticity. by William Alan Day
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equations. Moreover, the brevity of the book (about 300 pages) verifies its nonencyclopædic character. Actually, the book is a distillation of much of the research of the author and her co-workers over a twenty year period. Thus her particular techniques of functional analysis which she (along with many others) developed to attack the basic existence and uniqueness theorems of second order equations are used throughout. These include a discussion of Sobolev spaces, embedding theorems, and applications to the Dirichlet problem for elliptic equations. Separate chapters show how the methods can be adapted for the initial-boundary value problem (as well as other problems) for parabolic and hyperbolic equations. There is also a brief chapter which indicates how extensions can be made to elliptic, parabolic, and hyperbolic systems. A final chapter on finite difference methods for the solution of partial differential equations occupies about one-third of the book. The basic estimates for differential operators, developed at the beginning by techniques due to the author, are used to obtain analogous estimates for difference operators. The emphasis here is on establishing existence and uniqueness theorems for solutions of differential equations as a limit of the corresponding solutions for consistent difference equations. In fact, there is little discussion of difference methods from the point of view of computation, and thus numerical techniques are essentially unrelated to the presentation here.

There are a few problems and supplementary remarks at the end of each chapter. The problems are usually hard, but the remarks are frequently illuminating.

The Russian edition of the book first appeared in 1973 and the English version of 1985 has few changes. The bibliography does not include developments in the intervening years, and there are now methods which supersede the treatment we find here. However, for an account of the author's research in second order equations and systems the reader will find this book interesting and readable.

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Heat Conduction Within Linear Thermoelasticity. By WILLIAM ALAN DAY. Springer–Verlag, New York, 1985. viii + 82 pp. $28.00. ISBN 0-387-96156-9. Springer Tracts in Natural Philosophy. Vol. 30.

"Mathematical Experiments" could have been quite a suitable subtitle for this little book, which presents a thorough analysis of some elementary problems of the equations of thermoelasticity.

The classical equation of heat conduction of Fourier is derived under the assumption that the heat conducting body is *rigid*, eliminating any interactions of thermal and mechanical effects. The more sophisticated theory of thermoelasticity allows for thermomechanical coupling—both stress and entropy may depend on the strain and the temperature—thus parts of a body may expand or accelerate when heated. A systematic account of this theory is given in D. E. Carlson's treatise *Linear Thermoelasticity*, in Handbuch der Physik, Bd. VIa/2, Springer, 1972.

Day's book contains a detailed analysis of some very specific problems of one-dimensional, homogeneous thermoelasticity. Most of the material collected in this volume originally appeared as a series of articles in various journals in the past 3 to 4 years. They deal with (a) the initial value problems for the heat conduction or the steady-state oscillations of the temperature in a thermoelastic material, (b) the relationship between the solutions of the thermoelastic equations and the classical heat conduction equations in large time, and (c) the extent to which the maximum principle of the standard parabolic equations generalizes to the equations of thermoelasticity.

The mathematics involved—Fourier series, energy estimates, maximum principles—is elementary and somewhat routine (for the worker in the field of differential equations, at any rate); however, the framing of the questions and many of the results are quite novel and present excellent case studies for developing one's intuition about thermomechanical phenomena.

The book also presents a wealth of possibilities for making new conjectures, posing open problems, and making extensions in various directions; e.g., one may generalize the results to higher dimensions or to more general types of boundary data; allow for inhomogeneities and anisotropy; obtain intermediate asymptotics for the results on the large time behavior; or allow for temperature dependence in the conductivity coefficient.

The book is well written, with almost a classical elegance, clear explanations, and is easy to follow. My one complaint about it is of the scarcity of discussions of the physical implications, consequences, interpretations, and limitations of the results. For instance, the only comment made on the hypothesis $0 \leqq a \leqq 1$ of Theorem 8 is that "the restriction is a physically realistic one." It is not clear whether the restriction is in some sense necessary or if it is a reflection of the inadequacy of the method of proof. Equally mysterious and unexplained are the restrictions appearing in Theorems 10 and 21.

I enjoyed reading the book and learned from it. I also plan to use some selected topics in my course "Mathematical Methods for Engineers."

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Group Representations and Special Functions. By ANTONI WAWRZYNCZYK. D. Reidel, Dordrecht, the Netherlands, 1984. xvi + 688 pp. $119.00. ISBN 90-277-1269-7. Examples and Problems prepared by ALEKSANDER STRASBURGER. Translated by BOGDAN ZIEMIAN.

The relationship between special functions and group representations is a theme that goes back a long time. Given a group G and a homogeneous space X for it, the "special" functions are those defined on X which transform according to an irreducible representation of G under its action; and the central problem of harmonic analysis may be described (loosely) as that of expanding an "arbitrary" function on X as a "Fourier series" of those special functions. A variety of problems of analysis may be fitted into this very general framework: classical Fourier analysis, spherical harmonics, eigenfunction expansions and theory of modular forms, to mention a few. As the theory of group representations became more and more sophisticated, the scope of this point of view also becomes wider and deeper, and a long line of distinguished mathematicians have kept this theme active, such as E. Cartan, Weyl, Gel'fand, Harish-Chandra, Selberg and Langlands. Of course, the groups and homogeneous spaces have to be suitably restricted if a deep theory is to be developed. Experience has shown that the semisimple Lie groups and their homogeneous spaces offer the most profound context to pursue this theme. If G is such a group, it has a maximal compact subgroup K, and one knows that in any irreducible unitary representation of G, the dimension of the subspace of K-invariant vectors is at most 1; if it is 1, the representation is called *spherical*, and the matrix element defined by a K-invariant unit vector is called a *zonal spherical function.* These are perhaps among the most beautiful of all special functions, and their theory, due to Godement, Gel'fand, and above all, Harish-Chandra, is the model and the standard against which every subsequent generalization is measured.