please write name legibly

Math 225, Fall 2025 **Quiz #9** **Name:** ____________________

Find the general solution of $\vec{x}\,' = A\vec{x}$, where $A = \begin{bmatrix} 4 & -6 \\ 1 & -1 \end{bmatrix}$.

Solution: [Like Exercises 10–15 of section 6.4]

We begin with solving the eigenvalue problem $A\vec{\xi} = \lambda\vec{\xi}$. The eigenvalues are solution to the algebraic equation $\det(A - \lambda I) = 0$. We have

$$A - \lambda I = \begin{bmatrix} 4-\lambda & -6 \\ 1 & -1-\lambda \end{bmatrix},$$

and therefore

$$\begin{aligned}\det(A - \lambda I) &= (4-\lambda)(-1-\lambda) + 6 \\ &= -4 - 4\lambda + \lambda + \lambda^2 + 6 = \lambda^2 - 3\lambda + 2 = (\lambda - 1)(\lambda - 2).\end{aligned}$$

We conclude that the eigenvalues are $\lambda_1 = 1$ and $\lambda_2 = 2$. Next, we calculate the corresponding eigenvectors, $\vec{\xi}^{(1)}$ and $\vec{\xi}^{(2)}$ by solving $(A - \lambda_1 I)\vec{\xi}^{(1)} = \vec{0}$ and $(A - \lambda_2 I)\vec{\xi}^{(2)} = \vec{0}$.

We have

$$A - \lambda_1 I = \begin{bmatrix} 4-\lambda_1 & -6 \\ 1 & -1-\lambda_1 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix},$$

and therefore $(A - \lambda_1 I)\vec{\xi}^{(1)} = \vec{0}$ takes the form

$$\begin{bmatrix} 3 & -6 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

which amounts to $\xi_1 = 2\xi_2$. We pick $\xi_2 = 1$, $\xi_1 = 2$ for the solution and arrive at

$$\vec{\xi}^{(1)} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

Similarly, we have

$$A - \lambda_2 I = \begin{bmatrix} 4-\lambda_2 & -6 \\ 1 & -1-\lambda_2 \end{bmatrix} = \begin{bmatrix} 2 & -6 \\ 1 & -3 \end{bmatrix},$$

and therefore $(A - \lambda_2 I)\vec{\xi}^{(2)} = \vec{0}$ takes the form

$$\begin{bmatrix} 2 & -6 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

which amounts to $\xi_1 = 3\xi_2$. We pick $\xi_2 = 1$, $\xi_1 = 3$ for the solution and arrive at

$$\vec{\xi}^{(2)} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}.$$

Then, the DE's general solution, that is, $\vec{x}(t) = c_1 e^{\lambda_1 t}\vec{\xi}^{(1)} + c_2 e^{\lambda_2 t}\vec{\xi}^{(2)}$, takes the form

$$\vec{x}(t) = c_1 e^{t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 3 \\ 1 \end{bmatrix}.$$