please write name legibly

Math 225, Fall 2025 **Quiz #8** **Name:** ____________________

Apply the Convolution Theorem to solve the initial value problem $y'' + 4y' + 4y = f(t)$, $y(0) = 0$, $y'(0) = 0$, where $f(t)$ is an unspecified function.

Solution: [Like Exercises 17–19 of section 5.8]

Applying the Laplace transform to the equation, we get

$$\left[s^2\mathscr{L}\{y(t)\} - sy(0) - y'(0)\right] + 4\left[s\mathscr{L}\{y(t)\} - y(0)\right] + 4\mathscr{L}\{y(t)\} = \mathscr{L}\{f(t)\}.$$

Substituting the initial conditions into this equation, we obtain

$$(s^2 + 4s + 4)\mathscr{L}\{y(t)\} = \mathscr{L}\{f(t)\},$$

and therefore

$$\mathscr{L}\{y(t)\} = \frac{s}{(s+2)^2}\mathscr{L}\{f(t)\}.$$

Considering that $\mathscr{L}\{t\} = \frac{1}{s^2}$, we have $\mathscr{L}\{te^{-2t}\} = \frac{1}{(s+2)^2}$, which leads to

$$\mathscr{L}\{y(t)\} = \mathscr{L}\{te^{-2t}\}\mathscr{L}\{f(t)\}.$$

Then according to the Convolution Theorem, we have $y(t) = \left(te^{-2t}\right) * f(t)$, that is,

$$y(t) = \int_0^t (t-\tau)e^{-(t-\tau)} f(\tau)\, d\tau,$$

or equivalently,

$$y(t) = \int_0^t \tau e^{-2\tau} f(t-\tau)\, d\tau.$$

Either form is acceptable for answer.