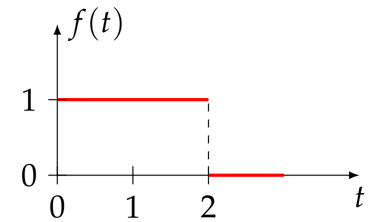
please write name legibly

Math 225, Fall 2025 **Quiz #7** **Name:** ____________________

1. Solve the initial value problem $y'' + y = f(t)$, $y(0) = 1$, $y'(0) = 0$, where $f(t)$ is shown in the graph.



Solution: [Like Exercises 4–12 of section 5.6]

Applying the Laplace transform to the equation, we get

$$\Big[s^2\mathscr{L}\{y(t)\} - sy(0) - y'(0)\Big] + \mathscr{L}\{y(t)\} = \mathscr{L}\{f(t)\}.$$

But we have $f(t) = 1 - u(t-2)$, and therefore $\mathscr{L}\{f(t)\} = \dfrac{1}{s} - \dfrac{1}{s}e^{-2s}$. Substituting this and the initial conditions into the equation, we obtain

$$(s^2+1)\mathscr{L}\{y(t)\} - s = \frac{1}{s} - \frac{1}{s}e^{-2s},$$

and therefore

$$\mathscr{L}\{y(t)\} = \frac{s}{s^2+1} + \frac{1}{s(s^2+1)} - \frac{1}{s(s^2+1)}e^{-2s}. \qquad (*)$$

Inverting this, calls for the partial fractions decomposition of

$$\frac{1}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1}. \qquad (**)$$

To find A, we multiply $(**)$ through by s:

$$\frac{1}{s^2+1} = A + s\frac{Bs+C}{s^2+1}.$$

and then set $s = 0$ and get $A = 1$. To find B and C, we multiply $(**)$ through by s^2+1:

$$\frac{1}{s} = (s^2+1)\frac{A}{s} + Bs + C,$$

and set $s = i$ which leads to

$$\frac{1}{i} = Bi + C.$$

Multiplying this by i yields

$$1 = -B + Ci.$$

We conclude that $B = -1$ and $C = 0$, and therefore $(**)$ takes the form

$$\frac{1}{s(s^2+1)} = \frac{1}{s} - \frac{s}{s^2+1}.$$

This enables us to express equation $(*)$ as

$$\mathscr{L}\{y(t)\} = \frac{s}{s^2+1} + \left[\frac{1}{s} - \frac{s}{s^2+1}\right] - \left[\frac{1}{s} - \frac{s}{s^2+1}\right]e^{-2s},$$

that is

$$\mathscr{L}\{y(t)\} = \mathscr{L}\{\cos t\} + \mathscr{L}\{1-\cos t\} - \mathscr{L}\{1-\cos t\}e^{-2s}.$$

We apply the inverse Laplace transform to this, and observe that on the right-hand side, the inverse transform of the 3rd term is the delayed version of that of the 2nd term:

$$y(t) = \cos t + \Big[1-\cos t\Big] - u(t-2)\Big[1-\cos(t-2)\Big],$$

which simplifies to

$$y(t) = 1 - u(t-2)\Big[1-\cos(t-2)\Big].$$