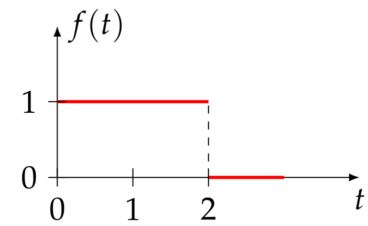
please write name legibly

Math 225, Fall 2025 **Quiz #6** **Name:** ____________________

1. (5 pts) Find the Laplace transform of the function $f(t)$ shown in the graph.



Solution: [Like Exercises 9–12 in section 5.1]

Since $f(t)$ is zero when $t > 2$, the evaluation of the Laplace transform reduces to an integration on the interval $0 \le t \le 2$. But $f(t) = 1$ on that interval, and therefore

$$\mathscr{L}\{f(t)\} = \int_0^2 e^{-st}\,dt = -\frac{1}{s}e^{-st}\Big|_{t=0}^{t=2} = -\frac{1}{s}e^{-2s} + \frac{1}{s}.$$

That's good enough, but optionally that may be expressed as

$$\mathscr{L}\{f(t)\} = \frac{1-e^{-2s}}{s}.$$

2. (5 pts) $\mathscr{L}\{te^{-3t}\sin t\} = ?$

Solution: [Like Exercise 11 in section 5.2]

It suffices to calculate $\mathscr{L}\{t\sin t\}$ because multiplying by e^{-3t} in the time domain amounts to shifting by -3 in the Laplace transform domain.

We have $\mathscr{L}\{\sin t\} = \frac{1}{s^2+1}$. Therefore

$$\mathscr{L}\{t\sin t\} = -\frac{d}{ds}\mathscr{L}\{\sin t\} = -\frac{d}{ds}\left(\frac{1}{s^2+1}\right) = \frac{2s}{(s^2+1)^2}.$$

We conclude that

$$\mathscr{L}\{te^{-3t}\sin t\} = \frac{2(s+3)}{\left[(s+3)^2+1\right]^2}.$$