please write name legibly

Math 225, Fall 2025 **Quiz #3** **Name:** ____________________

Find the general solution of each of the following differential equations:

(a) [3 pts] $2y'' + 3y' + y = 0$

(b) [3 pts] $y'' + 6y' + 10y = 0$

(c) [4 pts] $y'' + 4y' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 0$

Solution: [Like the exercises of sections 3.4 and 3.5]

(a) The roots of the characteristic equation $2r^2 + 3r + 1 = 0$ are

$$r = \frac{-3 \pm \sqrt{3^2 - 4 \times 2}}{4} = \frac{-3 \pm 1}{4},$$

and so we have two real roots, $r_1 = -\frac{1}{2}$, $r_2 = -1$. We conclude that the general solution is

$$y(x) = c_1 e^{-\frac{1}{2}x} + c_2 e^{-x}.$$

(b) The roots of the characteristic equation $r^2 + 6r + 10 = 0$ are

$$r = \frac{-6 \pm \sqrt{6^2 - 4 \times 10}}{2} = \frac{-6 \pm \sqrt{-4}}{2} = \frac{-6 \pm 2i}{2} = -3 \pm i.$$

We conclude that the general solution is

$$y(x) = e^{-3x}\left[c_1 \cos x + c_2 \sin x\right].$$

(c) The characteristic equation $r^2 + 4r + 4 = 0$ factors as $(r+2)^2 = 0$, and so we have one real root $r = -2$. We conclude that the general solution is

$$y(x) = c_1 e^{-2x} + c_2 x e^{-2x}.$$

Evaluating this at $x = 0$ and applying the initial condition $y(0) = 1$. results in $c_1 = 1$. To apply the second initial condition, we calculate the derivative

$$y'(x) = -2c_1 e^{-2x} + c_2\left[e^{-2x} - 2xe^{-2x}\right].$$

Evaluating this at $x = 0$ and applying the initial condition $y'(0) = 0$, we get

$$0 = -2c_1 + c_2.$$

Since $c_1 = 1$, it follows that $c_2 = 2$, and therefore the solution of the initial value problem is

$$y(x) = e^{-2x} + 2xe^{-2x},$$

which optionally may be factored as $y(x) = (1 + 2x)e^{-2x}$.