please write name legibly

Math 225, Fall 2025 **Quiz #2** **Name:** _____

A base jumper weighing $W = mg = 192\,\text{lb}$ (with equipment) leaps from a tall building. The parachute opens immediately. Assume that the air resistance coefficients is $\beta = 12\,\text{lb}/(\text{ft}/\text{sec})$ and the gravitational acceleration is $g = 32\,\text{ft}/\text{sec}^2$.

[5 pts] Explain how Newton's law of motion leads to an initial value problem for the downward velocity $v(t)$ as a function of time t;

[5 pts] Solve that initial value problem for $v(t)$.

Solution: [Like Exercises #12 of Section 2.6]

The forces acting on the diver are the weight W pulling downward, and the air resistance $\beta v(t)$ opposing the motion. Newton's law of motion says that mass times acceleration equals the sum resultant of forces acting on an object. Considering that the mass is $m = W/g$, and the initial velocity is zero, we arrive at the initial value problem

$$\frac{W}{g}v'(t) = W - \beta v, \quad v(0) = 0.$$

We rearrange the DE into the standard form

$$v'(t) + \frac{\beta g}{W}v(t) = g, \quad v(0) = 0,$$

and then plug in the given numerical data. We have $\frac{\beta g}{W} = \frac{12\times 32}{192} = 2$, and therefore

$$v'(t) + 2v(t) = 32, \quad v(0) = 0.$$

We multiply through by the integrating factor $\mu(t) = e^{2t}$ and obtain

$$\left(e^{2t}v(t)\right)' = 32e^{2t},$$

and integrate

$$e^{2t}v(t) = 16e^{2t} + C.$$

The initial conditions $v(0) = 0$ implies that $C = -16$. We thus get $e^{2t}v(t) = 16e^{2t} - 16$, and therefore

$$v(t) = 16\big(1 - e^{-2t}\big).$$