

Definition and properties of the Laplace transform

1. $\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt \equiv F(s)$
2. $\mathcal{L}\{e^{at} f(t)\} = F(s - a)$
3. $\mathcal{L}\{t f(t)\} = -\frac{d}{ds} F(s)$
4. $\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0) \quad | \quad \mathcal{L}\{f''(t)\} = s^2\mathcal{L}\{f(t)\} - sf(0) - f'(0)$
5. $\mathcal{L}\{u(t - a)f(t - a)\} = e^{-as}\mathcal{L}\{f(t)\} = e^{-as}F(s)$
6. $\mathcal{L}\{f(t) * g(t)\} = \mathcal{L}\{f(t)\} \mathcal{L}\{g(t)\} \quad \text{where} \quad f(t) * g(t) = \int_0^t f(t - \tau) g(\tau) d\tau$

Laplace transforms of a few specific functions

$$\begin{aligned} \mathcal{L}\{t^n\} &= \frac{n!}{s^{n+1}} & \mathcal{L}\{e^{at}\} &= \frac{1}{s - a} & \mathcal{L}\{u(t - c)\} &= \frac{e^{-cs}}{s} & \mathcal{L}\{\delta(t - c)\} &= e^{-cs} \\ \mathcal{L}\{\cos bt\} &= \frac{s}{s^2 + b^2} & \mathcal{L}\{\sin bt\} &= \frac{b}{s^2 + b^2} \end{aligned}$$