please write name legibly

Math 225, Fall 2024 **Quiz #9** **Name:** ____________________

1. (5pts) $\mathscr{L}\{t\cos 3t\} =?$

   Solution: [This is like Exercise 6 in section 5.2]

   We have $\mathscr{L}\{\cos 3t\} = \dfrac{s}{s^2+9}$. Therefore

$$\mathscr{L}\{t\cos 3t\} = -\frac{d}{ds}\mathscr{L}\{\cos 3t\} = -\frac{d}{ds}\Big(\frac{s}{s^2+9}\Big) = -\Big(\frac{1\cdot(s^2+9) - s\cdot 2s}{(s^2+9)^2}\Big)$$
$$= -\Big(\frac{9-s^2}{(s^2+9)^2}\Big) = \frac{s^2-9}{(s^2+9)^2}.$$

2. (5pts) $\mathscr{L}\{e^{-3t}\sin^2 t\} =?$

   Solution: [This is like Exercise 17 in section 5.2]

   It suffices to calculate $\mathscr{L}\{\sin^2 t\}$ because multiplying by e^{-3t} in the time domain amounts to shifting by -3 in the Laplace transform domain.

   To calculate $\mathscr{L}\{\sin^2 t\}$, we apply the trigonometric identity $\sin^2 t = \frac{1}{2}(1-\cos 2t)$. Then:

$$\mathscr{L}\{\sin^2 t\} = \frac{1}{2}\mathscr{L}\{1-\cos 2t\} = \frac{1}{2}\Big(\frac{1}{s} - \frac{s}{s^2+4}\Big) = \frac{1}{2}\Big(\frac{4}{s(s^2+4)}\Big) = \frac{2}{s(s^2+4)}.$$

   We conclude that

$$\mathscr{L}\{e^{-3t}\sin^2 t\} = \frac{2}{(s+3)\big[(s+3)^2+4\big]}.$$