Math 225, Fall 2024 **Quiz #8** **Name:** ____________________

A block weighing 64 pounds is suspended from a (long!) spring which in turn is suspended from the ceiling. The weight stretches the spring by two feet and comes to rest. The block is then set into motion by pulling it down by an additional 1 foot, imparting it a downward velocity of 4 ft/sec, and letting it go. Find the amplitude of the oscillations. Take $g = 32\,\text{ft/sec}$ for the gravitational acceleration.

4 points for setting up the initial value problem
4 points for solving the initial value problem
2 points for finding the amplitude

Solution: Let m be the mass of the block and k be the spring constant. We have

$$m = \frac{W}{g} = \frac{64}{32} = 2\,\text{slugs},$$
$$k = \frac{W}{\Delta x} = \frac{64}{2} = 32\,\text{lb/ft}.$$

Then the differential equation of motion, $m\ddot{u} + ku = 0$, takes the form $2\ddot{u} + 32u = 0$, that is, $\ddot{u} + 16u = 0$. The block's motion is the solution of the initial value problem

$$\ddot{u} + 16u = 0, \quad u(0) = 1, \quad \dot{u}(0) = 4.$$

The characteristic equation of the DE is $r^2 + 16 = 0$, which has roots $r = \pm 4i$. Therefore the general solution of the DE is

$$u(t) = c_1 \cos 4t + c_2 \sin 4t,$$

and consequently,

$$\dot{u}(t) = -4c_1 \sin 4t + 4c_2 \cos 4t.$$

Then

$$u(0) = 1 \quad \Rightarrow \quad 1 = c_1,$$
$$\dot{u}(0) = 4 \quad \Rightarrow \quad 4 = 4c_2$$

We conclude that $c_1 = 1$, $c_2 = 1$, and therefore the motion of the block is

$$u(t) = \cos 4t + \sin 4t.$$

The amplitude of the motion is

$$A = \sqrt{c_1^2 + c_2^2} = \sqrt{1^2 + 1^2} = \sqrt{2}.$$

Go to the next page for an interesting observation.

An observation with a moral

We are given the values for the weight $W = 64\,\text{lb}$, and the amount of stretch $\Delta x = 1\,\text{ft}$ that the weight produces. Let's forget about the numerical values for a moment and just use symbols for W and Δx in our solution. We have, as seen above, $m = \frac{W}{g}$ and $k = \frac{W}{\Delta x}$. Plugging these into the differential equation $m\ddot{u} + ku = 0$ we get

$$\frac{W}{g}\ddot{u} + \frac{W}{\Delta x}u = 0.$$

We see that W cancels and the equation simplifies to

$$\ddot{u} + \frac{g}{\Delta x}u = 0.$$

That tells us that the value of W *is immaterial in this problem*; it just drops out of the calculations! The prescription $W = 64\,\text{lb}$ has no purpose other than making you do some unnecessary arithmetic. The answer would have been the same if, for instance, you were given $W = 154.77\,\text{lb}$.

The moral of the story? Calculate with symbols to the extent possible, and account for numerical values only at the end.

This goes beyond this course—stick to symbolic calculations in your physics and engineering calculations. Keep numerical values for the end.