Math 225, Fall 2024 **Quiz #6** **Name:** ____________________

Solve the initial value problem $y'' + 2y' + 5y = 0$, $y(0) = 3$, $y'(0) = 1$.

Solution:
The characteristic equation is $r^2 + 2r + 5 = 0$ whose roots are

$$r = \frac{-2 \pm \sqrt{4 - 20}}{2} = \frac{-2 \pm 4i}{2} = -1 \pm 2i,$$

and therefore the general solution is

$$y(x) = e^{-x}\big(c_1 \cos 2x + c_2 \sin 2x\big). \qquad \text{(3 points)}$$

To apply the initial conditions, we calculate $y'(x)$:

$$y'(x) = -e^{-x}\big(c_1 \cos 2x + c_2 \sin 2x\big) + e^{-x}\big(-2c_1 \sin 2x + 2c_2 \cos 2x\big).$$

Plugging in the initial conditions we see that that

$$\begin{aligned} 3 &= c_1, \\ 1 &= -c_1 + 2c_2. \end{aligned}$$

We conclude that $c_1 = 3$, $c_2 = 2$, and therefore the solution of the initial value problem is

$$y(x) = e^{-x}\big(3 \cos 2x + 2 \sin 2x\big). \qquad \text{(7 points)}$$