please write name legibly

Math 225, Fall 2024 **Quiz #5** **Name:** ____________________

(a) [3 pts] Find the general solution of $4y'' + 4y' + y = 0$.

(b) [7 pts] Solve the initial value problem $4y'' + 4y' + y = 0$, $y(0) = 2$, $y'(0) = 0$.

Solution: [This is like Exercises #8 and 17 of Section 3.4]

(a) The characteristic equation is $4r^2 + 4r + 1 = 0$. This factors as $(2r + 1)^2 = 0$, and therefore $r = -1/2$. Alternatively, we may solve for r with the help of the quadratic formula:

$$r = \frac{-4 \pm \sqrt{4^2 - 4(4)(1)}}{8} = -\frac{4}{8} = -\frac{1}{2}.$$

We conclude that the general solution of the DE is

$$y(x) = c_1 e^{-x/2} + c_2 x e^{-x/2},$$

where c_1 and c_2 are arbitrary constants.

(b) To apply the initial conditions, we calculate the derivative

$$y'(x) = -\frac{1}{2} c_1 e^{-x/2} + c_2 \Big[e^{-x/2} - \frac{1}{2} x e^{-x/2} \Big],$$

and then evaluate $y(x)$ and $y'(x)$ at $x = 0$. We see that

$$\begin{aligned} y(x) &= c_1 e^{-x/2} + c_2 x e^{-x/2} && \text{and } y(0) = 2 && \Rightarrow && 2 = c_1, \\ y'(x) &= -\frac{1}{2} c_1 e^{-x/2} + c_2 \Big[e^{-x/2} - \frac{1}{2} x e^{-x/2} \Big] && \text{and } y'(0) = 0 && \Rightarrow && 0 = -\frac{1}{2} c_1 + c_2. \end{aligned}$$

It follows that $c_1 = 2$ and $c_2 = 1$. We conclude that

$$y(x) = 2e^{-x/2} + x e^{-x/2},$$

or equivalently,

$$y(x) = (x + 2) e^{-x/2}.$$