Math 225, Fall 2024 **Quiz #3** **Name:** ____________________

A skydiver weighing $W = mg = 192\,\text{lb}$ (with equipment) jumps from a plane. The parachute opens immediately.

(a) (2pts) Explain in a few words (and possibly a diagram) **how** Newton's Law of Motion leads to a differential equation for the parachutist's velocity.

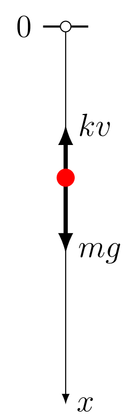
(b) (4pts) **Solve** the differential equation of motion to obtain the parachutist's velocity as a function of time.

(c) (4pts) How long does it take for the downward velocity to reach 90% of the limiting velocity?

The coefficient of air resistance is $k = 2\,\text{lb/ft/sec}$ and the gravitational acceleration is $g = 32\,\text{ft/sec}^2$.

Solution: [This is like Exercises #12 of Section 2.6]

(a) We take the x axis pointing down, with the origin set at the jumping point. The forces acting on the parachutist are (i) the force of gravity, mg (that is, his weight+equipment), which acts downward, and (ii) the force of air resistance, kv, which acts upward, as shown in this figure:



The resultant force acting on the parachutist is $mg - kv$, and that, according to Newton's Law of Motion, equals mass times acceleration:

$$m\frac{dv}{dt} = mg - kv.$$

(b) We rearrange this first order linear differential equation into the standard form

$$\frac{dv}{dt} + \frac{k}{m}v = g.$$

From $W = mg$ we get $192 = 32m$ and therefore $m = 6\,\text{slug}$. It follows that $\frac{k}{m} = \frac{2}{6} = \frac{1}{3}$, and the differential equation takes the form

$$\frac{dv}{dt} + \frac{1}{3}v = 32.$$

We determine the integrating factor

$$\mu(t) = e^{\int (1/3)\, dt} = e^{t/3},$$

and multiply the equation by $\mu(t)$,

$$e^{t/3}\frac{dv}{dt} + e^{t/3}\frac{1}{3}v = 32e^{t/3},$$

which then reduces to

$$\Big(e^{t/3}v(t)\Big)' = 32e^{t/3}.$$

Integrating this we obtain

$$e^{t/3}v(t) = 96e^{t/3} + c,$$

where c is the integration constant. We determine c by applying the initial condition $v(0) = 0$:

$$0 = 96 + c.$$

Therefore $c = -96$, and we arrive at

$$e^{t/3}v(t) = 96e^{t/3} - 96,$$

and finally

$$v(t) = 96 - 96e^{-t/3} = 96\Big(1 - e^{-t/3}\Big).$$

(c) The limiting velocity is

$$v_\infty = \lim_{t\to\infty} v(t) = 96\,\text{ft/sec}.$$

The time t when the velocity reaches 90% of the limiting velocity is obtained by solving

$$\frac{90}{100} \times 96 = 96\Big(1 - e^{-t/3}\Big),$$

which simplifies to $1 - e^{-t/3} = \frac{9}{10}$, or $e^{-t/3} = \frac{1}{10}$. To find t, we take the logarithm of both sides:

$$-\frac{1}{3}t = \ln\frac{1}{10} = -\ln 10,$$

and conclude that

$$t = 3\ln 10 \text{ seconds.}$$

Aside: That evaluates to approximately 6.9 sec.