Math 423, Spring 2024

1. Do the textbook's exercise 1.1.13.

Additionally: Figure 1.7 is not quite correct. The author forgot to specify scaling=constrained in Maple's plotting command. Make a proper plot of a cycloid.

2. When a circle rolls around a stationary circle without slipping, a point on the rolling circle's boundary traces an *epicycloid*. We saw in class how to derive the equation of the epicycloid and plot it, assuming implicitly that the circle rolls *outside* the stationary circle.

If the circle rolls *inside* the stationary circle, the curve traced is called a *hypocycloid*. Derive the parametric equation of the hypocycloid when the radii of the stationary and rolling circles are *a* and *b*, respectively. Plot one or more representative cases.

- 3. Do the textbook's exercise 1.1.17.
- 4. Plot the plane curve whose (signed) curvature is $\kappa(s) = s \sin s$, where *s* is the arclength parameter.

Hint: Let Let $\alpha(s) = \langle x(s), y(s) \rangle$ be the arclength parametrization of the curve, and let $\theta(s)$ be the angle that the curve's tangent vector makes with the positive *x* axis. We saw in class that *x*, *y*, and θ are related through the system of ODEs

$$\frac{dx}{ds} = \cos \theta(s),$$
$$\frac{dy}{ds} = \sin \theta(s),$$
$$\frac{d\theta}{ds} = \kappa(s).$$

Solve the system along with a suitable set of initial conditions, and then plot the curve $\alpha(s)$.

That system of ODEs has no solution in terms of elementary functions. You need to solve it numerically. See how that's done in maple-basics-2.mw in the course's web page.