Math 423, Spring 2024

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Please make an effort to *write neatly*, and *insert a few words* here and there to get your ideas across. It's difficult to understand (and evaluate) mathematics in the absence of guiding words.

You may write your solutions by hand, but a it would be terrific if you could do it in LATEX. In any case, upload your solutions to the course's site on Blackboard by the midnight of Wednesday February 7.

- 1. [3 pts] Let $f(x) = x^3 + 1$. Find the inverse function f^{-1} .
- 2. [3 pts] Let $a = \langle 1, 1, 1 \rangle$ and $b = \langle 1, 2, 3 \rangle$. Evaluate the cross product $a \times b$.
- 3. [3 pts] Find the equation of the tangent line to the parabola $y = x^2$ at x = 2.
- 4. [3 pts] *S* is the sphere of radius 1 centered at the origin the the *xyz* Cartesian coordinate system. Verify that the point *P* with coordinates $\left(\frac{\sqrt{3}}{4}, \frac{3}{4}, \frac{1}{2}\right)$ lies on the sphere. Find the equation of the tangent plane to the sphere at *P*.
- 5. [3 pts] In the adjacent diagram the radius of the circle is *a*. What is the length of *PH*?
- 6. [6 pts] Find the length of the parabolic arc $y = x^2$, $0 \le x \le 1$.
- 7. [6 pts] Find the length of the parametrically defined curve

$$\mathbf{r}(t) = \langle t - \sin t, 1 - \cos t \rangle, \quad 0 \le t \le 2\pi.$$

8. [6 pts] Consider the graph of $z = x^2 - y^2 + 5$ in the Cartesian coordinates. Find the surface area of the part of that graph that lies above the disk $x^2 + y^2 \le 1$.