for many small values of n, including 2, 3, 4, 5, 6, 8, 10, 11, 12, 15, 16, 17, 19, 20, 22, 24, and 30.

Both proofs that the a_is are relatively prime extend easily to the sequences obtained using any two relatively prime integers for a_1 and b_1, and recurrence pair $b_{i+1} = a_i b_{i-1}$, $a_{i+1} = b_{i+1} a_i^r + b_i a_{i-1}^s$ for any nonnegative integers r and s ($a_1 = \pm 1$ when $s > 0$).

ACKNOWLEDGMENTS We thank Professor Peter Hilton for his encouragement and suggestions on the manuscript.

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Another Simple Proof of $1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots=\frac{\pi^2}{6}$

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Recently Josef Hofbauer [2] shared with the readers of this MONTHLY a simple proof that

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots=\frac{\pi^2}{6}.$$

The crux of his proof was a clever application of Tannery's theorem. In this paper I use Fubini's theorem (twice) to prove this identity.

As has been frequently noted, it is enough to prove that

$$\sum_{n=1}^{\infty}\frac{1}{(2n+1)^2}=\frac{\pi^2}{8}.$$

Let us begin with the following double integral:

$$\int_0^\infty\int_0^1\frac{x}{(x^2+1)(x^2z^2+1)}dz\,dx=\int_0^\infty\left[\frac{\tan^{-1}xz}{x^2+1}\right]_0^1 dx$$
$$=\int_0^\infty\frac{\tan^{-1}x}{x^2+1}dx=\frac{\pi^2}{8}.$$

By Fubini's theorem this integral is equal to:

$$\int_0^1 \int_0^\infty \frac{x}{(x^2+1)(x^2z^2+1)}\,dx\,dz = \int_0^1 \int_0^\infty \frac{1}{2(z^2-1)} \left[\frac{2xz^2}{x^2z^2+1} - \frac{2x}{x^2+1}\right] dx\,dz$$
$$= \int_0^1 \frac{1}{2(z^2-1)} \left[\ln\left(\frac{x^2z^2+1}{x^2+1}\right)\right]_0^\infty dz$$
$$= \int_0^1 \frac{\ln z^2}{2(z^2-1)}\,dz$$
$$= \int_0^1 \frac{\ln z}{z^2-1}\,dz.$$

We now integrate this last integral by parts using $u = \ln z$ and $dv = dz/(z^2-1)$ to obtain:

$$\int_0^1 \frac{\ln z}{z^2-1}\,dz = \left[-\ln z \tanh^{-1} z\right]_0^1 - \int_0^1 \frac{-\tanh^{-1} x}{z}\,dz = \int_0^1 \frac{\tanh^{-1} z}{z}\,dz.$$

Finally, we use the McLaurin series expansion for $(\tanh^{-1} z)/z$ and then interchange this summation with the integral (Fubini again!):

$$\int_0^1 \frac{\tanh^{-1} z}{z}\,dz = \sum_{n=0}^{\infty} \int_0^1 \frac{z^{2n}}{2n+1}\,dz = \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}.$$

Putting the two ends together, we have our result. For readers who would enjoy seeing more proofs, Dan Kalman has given us "Six Ways to Sum a Series" [**3**] and Robin Chapman [**1**] has found fourteen ways to evaluate $\zeta(2)$ (not including Hofbauer's).

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