

Math 404, Fall 2020

Homework #9

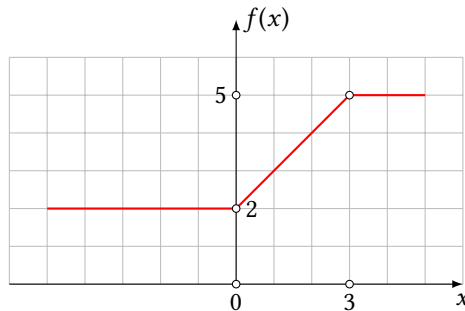
This homework concerns the initial value problem of traffic flow:

$$\begin{aligned} \frac{\partial \rho}{\partial t} + c(\rho) \frac{\partial \rho}{\partial x} &= 0 & -\infty < x < \infty, \quad t > 0, \\ \rho(x, 0) &= f(x) & -\infty < x < \infty, \end{aligned}$$

where $\rho(x, t)$ is the traffic density at the location x at time t , and

$$c(\rho) = u_{\max} \left(1 - \frac{2\rho}{\rho_{\max}} \right).$$

Let $\rho_{\max} = 8$, $u_{\max} = 4$, and $f(x)$ be as shown:



1. (8 points) Find the expression for the solution $\rho(x, t)$.
2. (3 points) Sketch the graph of $\rho(x, 1/2)$.
3. (3 points) Sketch the graph of $\rho(x, 1)$.
4. (3 points) Sketch the graph of $\rho(x, 3)$.

Don't just write down equations. Please explain what you are doing.