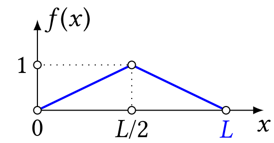
Math 404, Fall 2020
Homework #6

1. (5 points) Consider the initial boundary value corresponding to the vibration of a stretched string:

$$
\begin{aligned}
&u_{tt} = c^2 u_{xx} && 0 < x < L, \quad t > 0,\\
&u(0,t) = 0 && t > 0,\\
&u(L,t) = 0 && t > 0,\\
&u(x,0) = f(x) && 0 < x < L,\\
&u_t(x,0) = g(x) && 0 < x < L.
\end{aligned}
$$

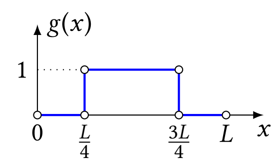


Take $f(x)$ as shown, and $g(x) = 0$. Express the solution $u(x,t)$ as a Fourier sine series. Set up the integrals that give the coefficients of that series but don't evaluate the integrals (unless you really want to) because their calculation is rather messy.

PS: That said, things simplify quite a bit after the dust settles. Here, for your information, is the solution of the problem, just to show that I calculated it to the end:

$$u(x,t) = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{\sin \frac{n\pi}{2}}{n^2} \sin \lambda_n x \cos \lambda_n c t.$$

2. (5 points) Repeat the previous problem when $f(x) = 0$ and $g(x)$ is as shown, but this time evaluate the integrals completely.



3. (a) (3 points) Apply d'Alember's formula to express the solution of the problem

$$
\begin{aligned}
&u_{tt} = c^2 u_{xx} && -\infty < x < \infty, \quad t > 0,\\
&u(x,0) = 0 && -\infty < x < \infty,\\
&u_t(x,0) = \sin x && -\infty < x < \infty,
\end{aligned}
$$

   (b) (3 points) Verify that $u(x,t) = \frac{1}{c} \sin x \sin ct$ is also a solution of that problem. Does that problem really have two solutions?