

Math 404, Fall 2020
Homework #5

We don't have "exams" in this course but this homework comes close to being an exam since it brings together just about everything that we have done so far in this semester. In that spirit, this homework will be graded more stringently than the other ones.

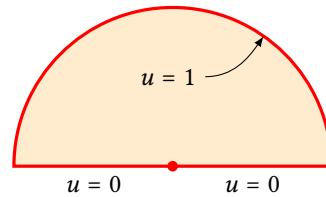
I expect to see words/explanations of what you are doing. Look at our online slides, or the textbook for that matter, to see that mathematical writing is more than merely equations/formulas strewn over a sheet. There is *a lot of text* that explains how one goes from one step to the next. It would be difficult, if not impossible, to follow the line of thought if that text were removed.

Many of the homework submission that I receive come with admirable explanations/text that makes them very pleasant to read and comprehend. Regrettably, some assignments are turned in with no text whatsoever :- (Attempting to read those is akin to watching a movie with the audio turned off—it is possible to sort of guess what is going on, but it is certainly not the way things are supposed to be. If yours are among the latter category, please make an effort to better present your work.

Sadly, in these days of pandemic and social isolation, I don't have the opportunity to meet and get to know you in person. All I see of you is what you hand in. In that sense, ***you are what you write!*** Take pride in your work and present it as best as it deserves. *Your work represents you!*

Rouben Rostamian

1. (5 points) Formulate the boundary value problem for the steady-state temperature $u(r, \theta)$ in polar coordinates in a semicircular lamina of radius 1. The temperature is prescribed as zero on the flat base and 1 around the curved edge.



2. (10 points) Look for solutions of the form $u(r, \theta) = R(r)\Psi(\theta)$. Give the details of the separation the variables procedure, including your analysis of the negative, zero, and positive values of the separation constant. Calculate the solutions $R_n(r)$ and $\Psi_n(\theta)$.
3. (2 points) Express the candidate for the solution of the boundary value problem as an infinite sum of the separated solutions $R_n(r)\Psi_n(\theta)$.
4. (4 points) Apply the boundary condition on the circular edge to determine the coefficients in that infinite sum.
5. (2 points) Write out the first five terms of the series solution that you have calculated.

Here is what the temperature distribution looks like (not required):

