Math 404, Fall 2020
Homework #3 (version 2)

1. (3 points) From Slide 29: Show that for any integers m and n we have

$$\int_0^L \sin \lambda_m x \sin \lambda_n x \, dx = \begin{cases} 0 & \text{if } m \neq n \\ L/2 & \text{if } m = n \end{cases} \qquad \text{where } \lambda_n = \frac{(2n-1)\pi}{2L}.$$

2. (5 points) Textbook, page 56, problem #1 (but I have changed the initial condition from $u(x,0) = x$ to $u(x,0) = 1$ to make solving it a little easier.

$$\begin{aligned} & u_t = u_{xx} && 0 < x < 1, \quad t > 0 \\ & u(0,t) = 0 && t > 0 \\ & u_x(1,t) = 0 && t > 0 \\ & u(x,0) = 1 && 0 < x < 1 \end{aligned}$$

3. (8 points) Solve the initial/boundary value problem (24) on Slide #44:

$$\begin{aligned} & \frac{\partial u}{\partial t} = \kappa \frac{\partial^2 u}{\partial x^2} && 0 < x < L, \quad t > 0 \\ & u(0,t) = 0 && t > 0 \\ & u(L,t) = \sigma \sin \omega t && t > 0 \\ & u(x,0) = 0 && 0 < x < L \end{aligned}$$