Math 404, Fall 2020
Homework #1 (on prerequisite materials)

Some of these exercises take more effort than others. To account for that, exercises 1–9 are worth 1 point each, 10–14 are worth 2 points each, and 15-18 are worth 5 points each.

Throughout these questions, the symbols a, b, c, d, A, B, C, D, and λ are constants.

1. Sketch the graphs of e^x and e^{-x} together in one coordinate system.
2. Sketch the graphs of $\sin x$, $\sin 2x$, and $\sin 3x$ together in one coordinate system over the interval $x \in (0, \pi)$.
3. Evaluate $\frac{d}{dx} e^{ax}$.
4. Evaluate $\frac{d^n}{dx^n} \sin ax$ for $n = 1, 2, 3, 4$.
5. The equation $\sin x = 0$ has infinitely many roots. What are they?
6. The equation $\cos x = 0$ has infinitely many roots. What are they?
7. What are the definitions of $\sinh x$ and $\cosh x$?
8. Sketch the graphs of $\sinh x$ and $\cosh x$ together in one coordinate system.
9. Apply the definitions of $\sinh x$ and $\cosh x$ in Exercise 7 to show that
$$\frac{d}{dx} \sinh ax = a \cosh ax, \qquad \frac{d}{dx} \cosh ax = a \sinh ax.$$
10. Show that the expression $Ae^{ax} + Be^{-ax}$ may be cast into the form $C \sinh ax + D \cosh ax$. What are C and D in terms of A and B?
11. How do you go about finding the general solution $y(x)$ of the differential equation $ay'' + by' + cy = 0$?
12. Apply the method of Exercise 11 to solve the differential equation $y'' + y' + y = 0$.
13. Apply the method of Exercise 11 to solve the differential equation $y'' + \lambda^2 y = 0$.
14. Apply the method of Exercise 11 to solve the differential equation $y'' - \lambda^2 y = 0$.
15. Evaluate $\int x \sin ax \, dx$. Show the steps of the calculation.
16. Let $f(x) = \frac{1}{2} - \left| x - \frac{1}{2} \right|$, or equivalently $f(x) = \begin{cases} x & \text{if } x < 1/2, \\ 1 - x & \text{if } x > 1/2. \end{cases}$

    Sketch a graph of $f(x)$ over the interval $(0, 1)$. Evaluate $\int_0^1 f(x) \sin(ax) \, dx$. Show the steps of the calculation.
17. Evaluate $\int \sin ax \sin bx \, dx$. Show the steps of the calculation.
18. Show that for all integers m and n we have:
$$\int_0^L \sin \frac{m\pi x}{L} \sin \frac{n\pi x}{L} \, dx = \begin{cases} 0 & \text{if } m \neq n, \\ L/2 & \text{if } m = n. \end{cases}$$