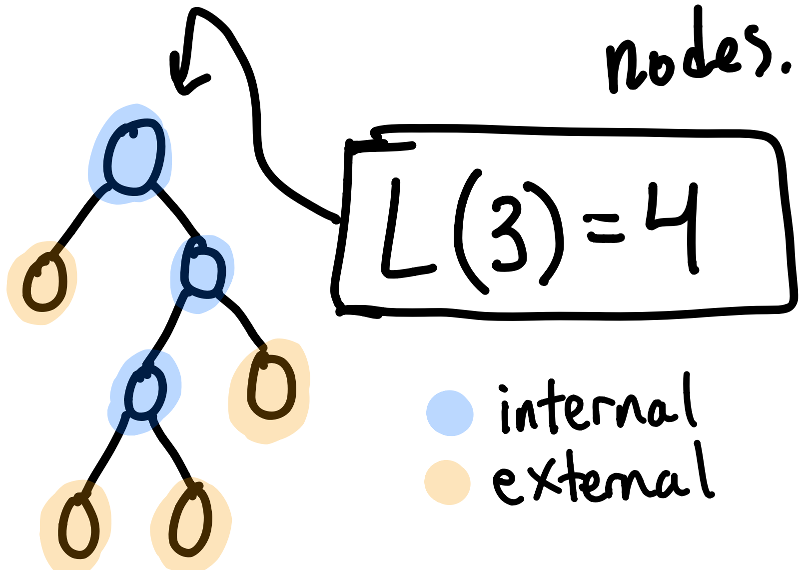
Proof by induction on a Full Binary Tree.

Show that $\boxed{i+1=e}$

$i \rightarrow$ # of internal nodes (w/ children)

$e \rightarrow$ # of external nodes (leaves)

Define $L(n)$: # of external nodes in a FBT with n internal nodes.



<u>Induct on n!</u>

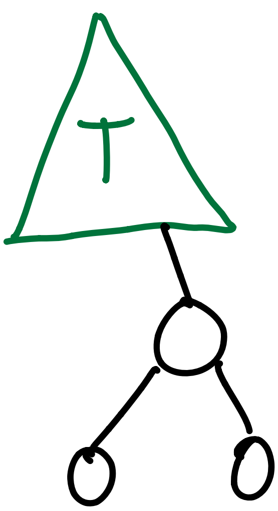
Base case:

The smallest FBT we can construct is comprised of a single node.

it has no internal nodes
it has one external node.

$$L(0) = 0 + 1$$
$$= 1$$
✓

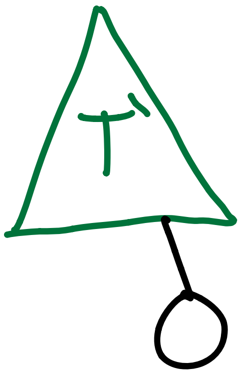
Assume that $L(x) = x + 1 \ \forall x < n$



Suppose you have a FBT of n internal nodes labeled T.

} here's a window into some leaves of T

If you remove two sibling leaves of T, you would be left with a different tree T'.



T' has two fewer leaves and one fewer internal nodes

a window into the change from T.

If T had n internal nodes, then T' has n-1. $(n-1 < n)$

By the induction hypothesis:

$$L(n-1) = (n-1)+1 = n$$

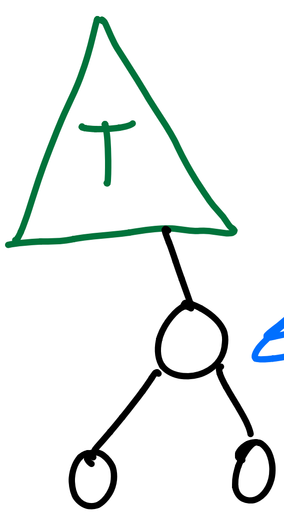
So T' has n leaves.

If we rebuild T from T', we just have to add in the leaves we had removed.

with n internal nodes in T

no change in rest of tree

$$L(n) = \#\text{ leaves in } T' + 2 - 1$$



was a leaf in T'
now internal

new leaf

$$L(n) = L(n-1) + 2 - 1$$
$$= n + 2 - 1$$
$$= n + 1$$

✓