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Centripetal forces in spatial voting: On the size of the Yolk*

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Abstract. The yolk, the smallest circle which intersects all median lines, has been shown to be an important tool in understanding the nature of majority voting in a spatial voting context. The center of the yolk is a natural 'center' of the set of voter ideal points. The radius of the yolk can be used to provide bounds on the size of the feasible set of outcomes of sophisticated voting under standard amendment procedure, and on the limits of agenda manipulation and cycling when voting is sincere. We show that under many plausible conditions the yolk can be expected to be small. Thus, majority rule processes in spatial voting games will be far better behaved than has commonly been supposed, and the possible outcomes of agenda manipulations will be generally constrained. This result was first conjectured by Tullock (1967).

1. Introduction

Recent advances in social choice theory in the spatial context have shown that a new geometric construct, the yolk – i.e., the smallest circle (or sphere or hypersphere) that intersects all median lines (or hyperplanes) – offers important insights into the nature of majority rule processes when individual preferences are Euclidian. The size of the yolk (measured by its radius r) measures how close the majority preference structure is to having a majority winner or

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core outcome (McKelvey, 1986).[1] In particular, the size of the uncovered set is determined by the size of the yolk. Also, if the center of the yolk is a feasible outcome, the set of outcomes given sophisticated voting under amendment procedure must be contained within a circle with radius of 4r around the center of the yolk. And the outcome of competition between candidates rationally seeking electoral victory must be contained in the same circle. (See McKelvey, 1986; Feld et al., 1987; cf. Miller, 1980; Banks, 1985; Shepsle and Weingast, 1984.)[2] Other recent important results are (1) that the likely outcomes of most reasonable probabilistic choice processes for generating agendas are concentrated within 4r of the center of the yolk (Ferejohn, McKelvey and Packel, 1984); (2) that no alternative x can lose to any alternative which is more than 2r further away from the center of the yolk than x is (McKelvey, 1986; cf. Feld, Grofman and Miller, 1989 forthcoming); (3) that agendas which generate movement toward the center of the yolk are far easier to create than those which move away from the yolk (Feld and Grofman, 1987a); and (4) that the range of outcomes which can be generated by one-issue-at-a-time decision making can be bounded by a simple function expressed in terms of the yolk (Feld and Grofman, 1988b forthcoming).[3] Many of these results were first conjectured by Tullock (1967) in a much neglected paper.

Because the size of the yolk is critical to understanding the dynamics of majority rule in the spatial context, knowing the conditions under which the yolk can be expected to be small (relative to the distribution of voter ideal points) is of great importance. If the yolk is small, then outcomes of most voting processes will be confined to a small, centrally located region of the space. Thus, the potential domain of agenda manipulation (McKelvey, 1976, 1979; Schofield, 1978; cf. Shepsle, 1979; Riker, 1982) will be severly confined.

In this paper we shall provide theoretical results which provide bounds for the yolk; and which indicate that the yolk will typically be quite small relative to the distribution of voter ideal points.

A principal theoretical result is an analogue to the Plott (1967) symmetry conditions. We show that when we can pair enough voter ideal points on either side of some circle so that the paired points plus the points contained within the circle constitute a majority of all voter ideal points, then the radius of the circle is an outer bound for the radius of the yolk. It follows that the smallest circle containing any minimal winning coalition is an outer bound for the size of the yolk. We also show that the yolk is always smaller than such a circle because, in w dimensions, the interior of the yolk can contain no more than $[(w-1)n - (w+1)] / 2w$ of the n voter ideal points. Thus, in two dimensions, at least 3/4ths of the voter ideal points must lie outside the yolk.

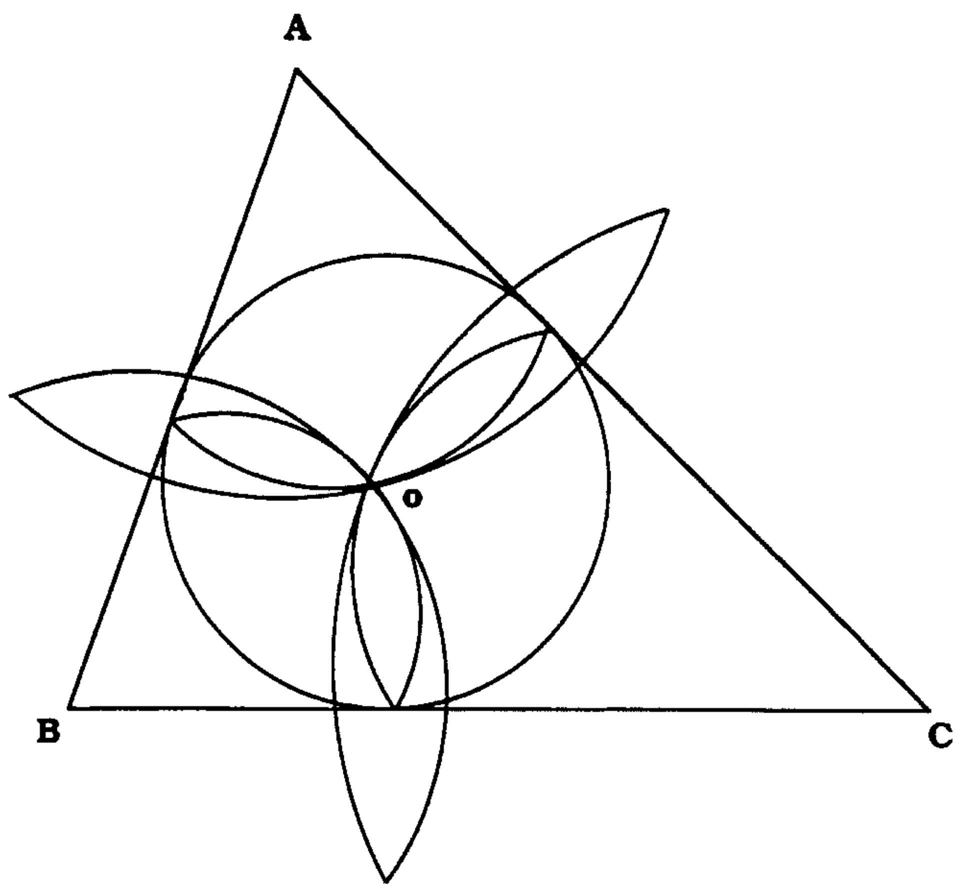

Figure 1. The petals of the win-set and half-win set around o, the center of the yolk

2. Theoretical results in the size of the yolk

The results we give are based on Euclidean preferences, i.e., voters are assumed to have a most preferred point in the space and to prefer alternatives closer to their ideal point to those further away. This is a common assumption which simplifies both exposition and proofs (McKelvey, 1986).[4] We assume that the number of voters, n, is odd. While our results hold for any number of dimensions unless otherwise specified, our illustrations and terminology will be two-dimensional. Thus we will refer to circles, lines, etc., rather than hyperspheres, hyperplanes, etc. Proofs of all lemmas and theorems are provided in a mathematical Appendix.

Definition 1: The *win set* of any point is the set of alternatives that are preferred to it by a majority of the voters.

Definition 2: The *half-win set* of any point, x, is the set of points along any ray from x one half the distance from x to the boundary of its win set along that ray, i.e., to the median line perpendicular to the ray.

We show in Figure 1 the win-set and half-win set of a point for a simple three voter example.

Definition 3: The *yolk* is the smallest circle that intersects all median lines.

It follows that the yolk is the smallest circle surrounding the half-win set of any point. This equivalence is established by the fact the limit in each direction of the win-set of a point is just twice the distance to the median line in that direction. Consequently, a circle around a point intersects all median lines if and only if twice that circle surrounds the win-set of the point.

We begin with some loose bounds on the size of the yolk, and then proceed to establish some narrower constraints.

Theorem 1: The radius of the yolk is at most one half of the radius of any circle which includes all voter ideal points.

We can proceed to consider limits on the size of the yolk that follow from the concentration of points. It is easy to see that any majority of points provides an outer bound for the size of the yolk.

Theorem 2: The radius of a circle including a majority of the voter ideal points (a minimal winning coalition) provides an outer bound for the radius of the yolk.

This 'bound' is almost certainly much greater than the size of the yolk since the next theorem indicates that in two dimensions the yolk can include no more than $1/4$ of the voter ideal points.

Theorem 3: In two dimensions, the interior of the yolk can contain or touch no more than $(n-3)/4$ of the voter ideal points, where n is the number of voters. More generally, in a spatial voting game with w dimensions and n voter ideal points, the yolk can contain or touch no more than $[(w-1)n-(w+1)]/2w$ of the ideal points.

When the approximate location of the center of the yolk can be determined, this theorem provides bounds for the size of the yolk; in two dimensions we know that the yolk can be no bigger than a circle centered at that location including $1/4$ of the voter ideal points.

This theorem implies that, in two dimensions, for $n < 7$ there are no points in the yolk, for $n < 10$ there is at most one point in the yolk, etc. This suggests that circles containing a majority of voter ideal points are in fact likely to be much larger than the yolk.[5]

We now provide theorems that show that certain types of additions of voter ideal points can only decrease the size of the yolk, and demonstrate implications for bounds on the yolk.

Theorem 4: If a pair of voter ideal points is added to a set of voter ideal points such that the line segment connecting the pair passes through the original yolk, the resulting yolk can be no larger than for the original set.

If the original yolk is large, then it is easy to satisfy the conditions of Theorem 4. Of course as the yolk shrinks, the condition approximates the Plott

(1967) condition. Note, too, that even if the distribution of added points is very different from that required by the theorem, it is not necessarily true that the yolk expands in size. For example, if the added points lie entirely or disproportionately to one side of the old yolk, the symmetry condition certainly will not be met, and the new yolk almost certainly will not be contained in the old yolk, as its location will have shifted substantially. But the new yolk may still be smaller, in terms of its radius, than the old yolk.

Theorem 5: In two dimensions, if the set of voter ideal points can be partitioned such that some of the voter ideal points are in pairs whose line segments pass through a circle, and the majority of the rest of the voter ideal points are within that circle, then the circle provides a bound for the size of the yolk.

Theorem 5 follows directly from Theorems 3 and 4.

Theorem 5 is a potentially important result because it is often easy to find a centrally located circle including a dense concentration of voter ideal points that meets the condition given in that theorem, e.g., if voter ideal points are close to a multivariate normal distribution.

3. Discussion

As we have shown, the addition of points with any degree of symmetry around a 'central' location tends to reduce the size of the yolk. In general, if there is symmetry around a 'center,' there will tend to be a small yolk. In two dimensions, the bivariate normal distribution is most often used as the model for distributions of voter preferences in two dimensions. (See Chamberlin and Cohen, 1978; Merrill, 1984, 1985.) While such a model is only an approximation, it is easy to see that if an actual distribution is anything close to that approximation, the yolk will be small. If there were a perfect bivariate normal distribution, there would be a majority rule core, i.e., the radius of the yolk would be zero. If deviations from a perfect bivariate normal distribution are minimal, the yolk would be small. (Cf. Koehler, 1988; also see discussion of Theorem 3 in the Appendix.)

If the yolk were small enough, with a small finite number of alternatives, there would often be a Condorcet winner. Thus, our results help explain why previous authors (such as Chamberlin and Cohen, 1978: Table 4) who simulate voting in a spatial context by positing an underlying univariate normal distribution in each dimension, find a Condorcet winner from among a small subset of alternatives 100% (or nearly 100%) of the time. Although Tullock (1967) did not make use of the minimal circle touching all median lines but thought, rather, in terms of the minimal region which was at the intersection of all median lines, this is effectively the result he posited.

Appendix

Proof of theoretical results on the size of the yolk

Let r_p designate the radius of the smallest circle that includes every voter ideal point. Thus, r_p is a rough measure of the size of the Pareto set.

Theorem 1. In two dimensions, the radius of the yolk is at most one half of the radius of the circle including all voter ideal points, i.e., it must be the case that $r \leq r_p/2$.

Proof: There always exist three median lines that are tangent to the yolk, forming a triangle within which the yolk is inscribed (see Figure 2). By definition of a median line, at least $(n+1)/2$ ideal points must lie on or outside each line. Thus at least one ideal point must lie on or outside both of each pair of median lines, i.e., in each of the shaded regions (and including the boundaries thereof) in Figure 2, since any two majorities must have at least one member in common. Consequently, the triangle formed by the three median lines is contained in the convex hull of these three points and thus within the convex hull of all ideal points, i.e., the Pareto set PO(X), and finally within any circle circumscribing PO(X). It is a well-known result in plane geometry that, for any triangle, the radius of the inscribed circle can be no more than one half of the radius of the circumscribed circle; this maximum occurring when the triangle is equilateral. Q.E.D.

In proving the theorem, we also proved the weaker result that in two dimensions, the yolk is properly contained in the Pareto set. The bound on the size of the yolk established by Theorem 1 is a very weak one. In two dimensions, the radius of the yolk r can equal $r_p/2$ only if there are just three ideal points forming an equilateral triangle, or all ideal points are 'piled on top of each other' at just three locations (with fewer than half of the ideal points at any one location) likewise forming an equilateral triangle. The yolk can touch the boundary of the Pareto set only if there are just three ideal points, or (as above) all ideal points are located at just three locations, or if all ideal points are collinear (in which case PO(X) is a line segment and the yolk is a point on the line).

Let r_{min} designate the radius of the smallest circle that includes some set of $(n+1)/2$ voter ideal points, i.e., a minimum winning coalition under simple majority rule. Then we have the following.

Theorem 2. The radius of the yolk is no greater than the radius of the smallest circle that includes the ideal points of some minimal winning coalition, i.e., it must always be the case that $r \leq r_{min}$.

Proof: This is immediate because no median line can fail to intersect any circle including the ideal points of any (minimal) winning coalition, for otherwise at least $(n+1)/2$ ideal points would lie to one side of the line. Thus the yolk is some other smaller (and intersecting) circle. Q.E.D.

Theorem 3. In two dimensions no more than $(n-3)/4$ of the voter ideal points can lie in the interior of the yolk.

In words, fewer than one quarter of the voter ideal points can lie in the interior of the yolk.

Proof: There are at least three median lines tangent to the yolk. These three lines partition the space into seven regions, and thus also the set of ideal points into seven subsets, as shown in Figure 3.

Because some ideal points lie on the lines, we must define the partition carefully. For present purposes, it is convenient to define the sets A, C, and E as including the lines that form their boundaries, the set G as excluding the lines that form its boundaries, and thus the sets B, D, and F as including their boundaries with G but excluding their other boundaries. Let the number of voter ideal points in each region be given by the corresponding lower case letter. Thus:

$$n = a + b + c + d + e + f + g. \tag{1}$$

Since each line is a median line, each can have at most $(n-1)/2$ ideal points on either side of it. Thus each has at least $(n+1)/2$ ideal point on and to one side of it. Thus:

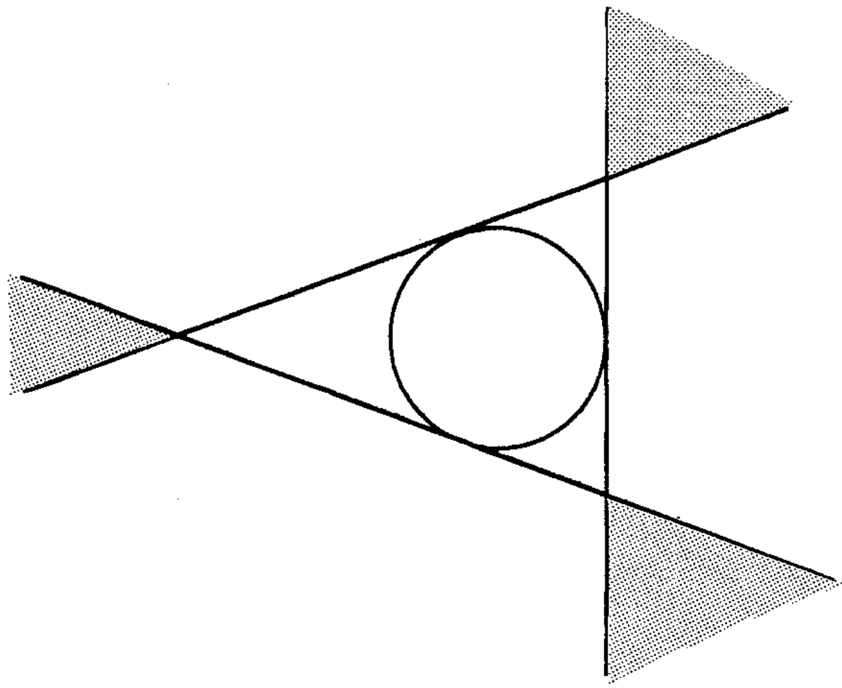

Figure 2. Median triangle bounding the yolk

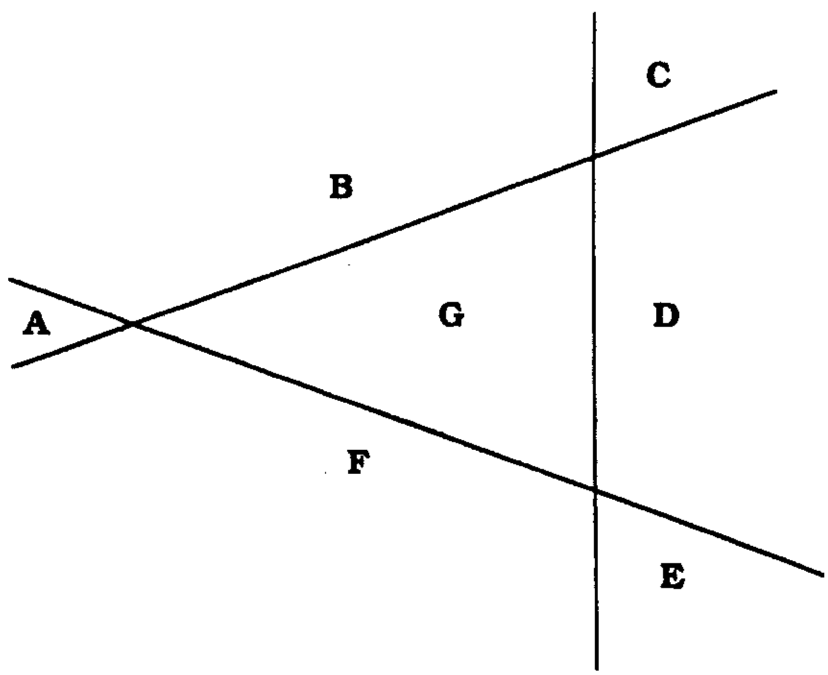

Figure 3. Construction used to prove Theorem 3

$$a + b + c \geq (n+1)/2;$$
$$c + d + e \geq (n+1)/2;$$
$$e + f + a \geq (n+1)/2.$$

Simplifying, adding the three inequalities, and substituting in (1) above, we get:

$$4a + 2b + 4c + 2d + 4e + 2f-3 \geq 3n = 3a + 3b + 3c + 3d + 3e + 3f + 3g$$

and

$$g \leq [a + c + e - 3 - (b + d + f)]/3. \quad (2)$$

Clearly g takes on its maximum value when $(b + d + f) = 0$. Thus:

$$g_{max} \leq [(a + c + e)/3] - 1.$$

But also:

$$g_{max} = n - (a + c + e)$$

so

$$n - (a + c + e) \leq [(a + c + e)/3] - 1$$

and

$$(3n + 3)/4 \leq a + c + e.$$

Thus:

$$g_{max} = n - (a + c + e) \leq n - (3n + 3)/4 = (n-3)/4.$$

Since the interior of the yolk is included in G, the theorem is proved. Q.E.D.

Notice that this is an upper bound for the interior of the whole triangle G, so the interim of the yolk may contain even fewer points.

Theorem 3 implies that, in two dimensions, there must be at least seven voter ideal points before any can lie within the yolk.

Theorem 3 suggests quite powerfully that the yolk is small relative to the dispersion of ideal points when the ideal points are large in number and 'reasonably' distributed.

First, we have seen that the yolk is more or less centrally located. Second, if the ideal points are even very roughly normally distributed, they will tend to concentrate in the vicinity of the center of the distribution. Thus, even if the yolk were to include nearly one quarter of the ideal points, it would most likely probably be small, relative to the Pareto set.

But much more potent is the following consideration. If the ideal points are distributed even very roughly normally, nothing approaching one quarter of them could be within the yolk, for establishing the maximum given in Theorem 3 requires that the ideal points be distributed very oddly indeed. All ideal points not in G must be clustered in A, C, and E, with none at all in B, D, and E. To get a substantial number of ideal points within the yolk, there must be substantially more points in $A \cup C \cup E$ than in $B \cup D \cup F$. (See Expression (2)). Yet G is centrally located in the distribution of ideal points, and each of the other regions is essentially a pie-slice-shaped wedge radiating outward from this generalized median. While the six wedges certainly need not be the same size, as indicated by the magnitude of the angle between their boundaries, it may be readily checked that A ∪ C ∪ E and B ∪ D ∪ E are essentially the same size, in that the sum of the angles is the same, i.e., 180°, for both triples. We might reasonably expect, therefore, rather than $b + d + f = 0$ and $a + c + e = n - g$ (as we assumed in determining g_{max}), that much more typically $a + c + e \approx b + d + f$. Actually, this general expectation must be refined a bit. Given that g must be non-negative, it follows from expression (2) above that $b + d + f$ cannot exceed $a + c + e$ and, in fact, the latter sum must exceed the former by at least three. (This asymmetry results from the fact that the two triples of regions are not formed by any three lines coming together in the vicinity of the center of the distribution but specifically by three median lines.) But the expectation holds that typically $a + c + d$ would exceed $b + d + f$ by not many more than three points, which implies that typically no more than a few ideal points would be within the yolk. Moreover, as the number of ideal points increases, and if their diversity increases also (i.e., in the absence of any peculiar clustering of ideal points), we should expect $a + c + d$ and $b + d + f$ to become relatively more equal, and thus relatively fewer points to lie within the yolk. And, if, as the number of ideal points increases, the proportion lying within the yolk decreases while their overall dispersion remains about the same, it must be that the yolk is shrinking in size relative to the (more or less constant) dispersion of ideal points. This two-dimensional result can be generalized quite straightforwardly.

Theorem 3′. In a spatial voting game with w dimensions and n voter ideal points, the interior of the yolk can contain no more than

$$\frac{(w-1)n - (2+1)}{2w}$$

of the ideal points.

In two dimensions, there are (at least) three lines tangent to the yolk and forming what we may call a median triangle (with three sides and three vertices). (See Figure 2.) In three dimensions, there are (at least) four median planes tangent to the yolk and forming a median tetrahedron (with four faces and four vertices). In w dimensions, there are (at least) $w+1$ median hyperplanes (each $w-1$ dimensional) tangent to the yolk and forming a median 'hyper-hedron' (with $w+1$ 'faces' and $w+1$ vertices).

Let f_i be the number of ideal points on or beyond each face i (cf. b, d, and f in Theorem 3 above), v_j the number of ideal points beyond each vertex j (cf. a, c, e in Theorem 3 above), and c the number of ideal points inside the 'hyper-hedron.' Thus:

$$n = \Sigma f + \Sigma v + c \tag{1}$$

Then, for each median hyperplane, we have the inequality:

$$f_i + \Sigma v_j \geq (n+1)/2$$

where the summation is over all w vertices that lie in the median hyperplane that defines f_i.

Simplifying, adding up the $w+1$ inequalities, and substituting in (1), we get:

$$2\Sigma f + 2w\Sigma v - (w+1) = (w+1)\Sigma f + (w+1)\Sigma v + (w+1)c$$

and

$$c \leq (w-1)(\Sigma v - \Sigma f)/(w+1) - 1 \tag{2}$$

Clearly c takes on its maximum value c_{max} when $\Sigma f = 0$. Thus:

$$c_{max} \leq (w-1)\Sigma v/(w+1) - 1.$$

But also

$$c_{max} = n - \Sigma v$$

so

$$n - \Sigma v \leq (w-1)\Sigma v/(w+1) - 1$$

and

$$(w+1)(n+1)/2w \leq \Sigma v.$$

Thus

$$c_{max} = n - \Sigma v \leq [(w-1)n - (w+1)]/2w.$$

Since the interior of the yolk lies within the median 'hyper-hedron,' the theorem is proved.

If n is large, the maximum proportion of ideal points that may lie within the median 'hyper-hedron' is essentially $(w-1)/2w$. Thus, the maximum proportion increases with the dimensionality of the space and approaches a limit of 50%. For example:

w	*proportion*
2	1/4 = 25%
3	2/6 = 33.3%
4	3/8 = 37.5%
5	4/10 = 40%
6	5/12 = 41.7%
limit	50%

We now present a further theorem which implies that, under plausible conditions, the yolk shrinks in size as the number of voter ideal points increases.

Theorem 4. Given any set of n voter ideal points with a given yolk of radius r, if two voter ideal points are added such that the line between them intersects this yolk, the new yolk for the n+2 ideal points is no larger than the yolk.

Proof: Let r be the radius of yolk for the original n ideal points and let x_{n+1} and x_{n+2} be the two additional ideal points. See Figure 4. Suppose that the line through x_{n+1} and x_{n+2} passes through the old yolk, as shown in Figure 4.

If n is odd, there is exactly one median line passing through the space at a given angle (relative to some coordinate system). We consider any median line M for the distribution of n ideal points and consider whether it is still a median line in the distribution of n+2 points and, if not, where the median line M′ parallel to the old M lies in the new distribution.

There are two possibilities: (1) M intersects the line passing through x_{n+1} and x_{n+2} between x_{n+1} and x_{n+2}, or (2) it does not. If (1), one point has been added on either side of M and M remains a median line in the new distribution. If (2), two points have been added on the same side of M and most likely M is no longer a median line (though it might be if, fortuitously, two or more ideal points lie on M and fewer than $(n-1)/2$ ideal points previously lay on the side of M to which s_{n+1} and x_{n+2} were added). To find the new median line M′ parallel to M, we shift M toward

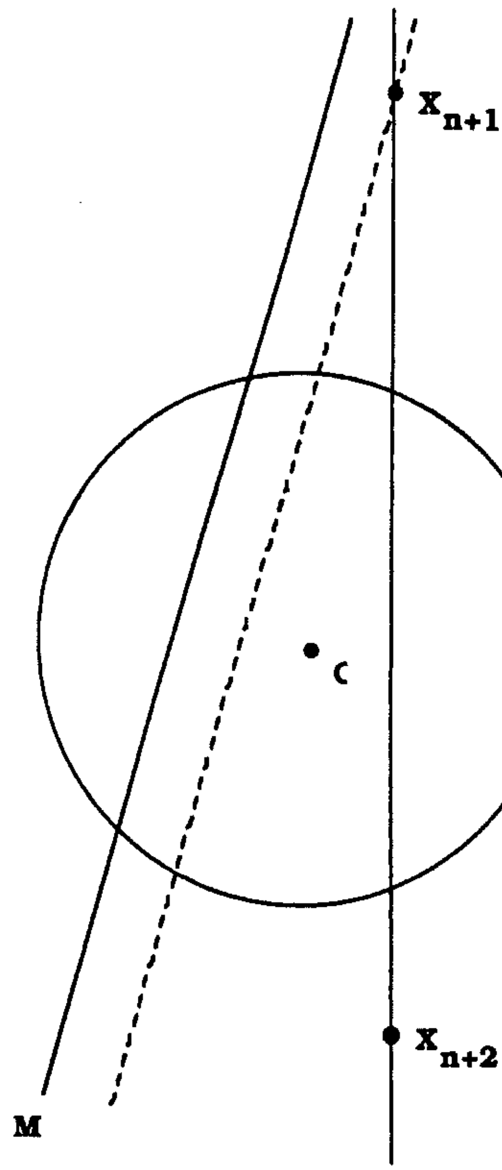

Figure 4. Construction used in proof of Theorem 4

x_{n+1} and x_{n+2} until we hit the first ideal point. Thus the furthest we could shift M would be until we hit the closer of x_{n+1} and x_{n+2} (e.g., x_{n+1} in Figure 3). Since M intersects the old yolk and the line through x_{n+1} and x_{n+2} intersects the old yolk, M′ must intersect the old yolk.

This argument can be repeated for every median line for the distribution of n ideal points. Thus every median line for the distribution of n + 2 ideal points passes through the old yolk and the new yolk is contained in the old yolk. Since some of the median lines have shifted, the containment may be strict, so that $r' < r$ (in which case probably $c' \neq c$ where c is the center of the old yolk and c′ is the center of the new yolk), but in any event $r' \leq r$, where r′ and c′ are the radius and center of the new yolk. Q.E.D.

Note that this theorem reflects a type of generalization of the well-known Plott (1967) symmetry conditions.

Theorem 5: In two dimensions, if the set of voter ideal points can be partitioned such that some of the voter ideal points are in m pairs whose line segments pass through a circle, and the majority of the rest of the voter ideal points are within that circle, then the circle provides a bound for the size of the yolk.

Proof: This result follows directly from Theorems 3 and 4. Start with the points that are not paired; a circle surrounding a majority of those points is an outer bound for the yolk of those points. All the other points can be paired, two at a time, so that the connecting lines pass through that circle. But the yolk of the combined set is no larger than the original yolk. Q.E.D.

Notes

1. The yolk will be of radius zero if and only if there is a majority winner.
2. This bound has been strengthened to 3.7r by Feld et al. (1987).
3. The same is true for the outcomes of possible committee jurisdictional assignments with a set of single issue-dimension committees operating under a germaneness rule (see also Krehbiel, 1984).
4. However, there is a straightforward way to extend all the results to deal with the more general case of convex preferences (see Cox, 1987; Feld and Grofman, 1987a).
5. For example, as w increases, the number of points that can be found interior to the yolk increases from 25% (for two dimensions) to 50% (in the limit). See Theorem 3′ in Appendix (cf. Schofield, Grofman and Feld, 1988).

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