BICAMERALISM AND THE THEORY OF VOTING
A Comment

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IN A RECENT article in this journal, Donald Gross (1982) attempted to extend the formal theory of voting to bicameral bodies. Gross correctly observes that the theory of voting pioneered by Black (1958) and Farquharson (1969) and extended by Miller (1977, 1980), McKelvey and Niemi (1978), Bjurolf and Niemi (1982), and others has focused almost exclusively on voting processes in simple unicameral bodies (but see Shepsle 1979). He further observes that, in point of fact, many legislative bodies (especially in the United States) are bicameral in structure. Finally, he argues that voting within a single chamber of a bicameral body cannot in general by analyzed as if that chamber were a unicameral voting body; rather explicit theoretical account must be taken of the fact that the chamber is part of a larger legislative system.

In sum, then, Gross attempts to extend the formal theory of voting in a way that is both empirically relevant and theoretically interesting. But unfortunately Gross's effort is severely flawed. In this commentary, I point out the most serious of these flaws and attempt to rectify them. In so doing, I use notation, assumptions, and examples employed by Gross; therefore this comment should be read with his article at hand.

The first point to make is that the structure of a bicameral voting process (at least the simplified versions considered by Gross) can be represented by a perfectly "proper" voting (or division) tree of the sort introduced by Farquharson (1969: 10ff) and illustrated by Gross in his Figures 1 and 2. That is, it is quite possible to avoid the messy construction devised by Gross and depicted in his Figure 3, which, as he admits, "is not, strictly speaking, a voting tree as traditionally utilized in the literature because a number of divisions do not represent actual votes" (p. 516) and which, indeed, does not even display the structure of a topological tree.

Figure 1 of this note displays the voting process for the example depicted by Gross in his Figure 3. Note that my Figure 1 is a proper voting tree, in that it displays the appropriate structure and every division of the tree represents an actual vote. It differs from a "traditional" voting tree, of course, in that different divisions are assigned to different sets of voters. Moreover, its divisional structure violates Farquharson's Axiom II (1969: 11) in that a subset of alternatives may be identical with the preceeding whole set. (One implication of this is that we cannot use the formal definition of sincere voting proposed by Farquharson [1969: 18]; rather sincere behavior must be defined substantively with respect the question being put to a vote at each division.)

NOTE: This article is a critical comment on Donald R. Gross's "Bicameralism and the Theory of Voting" which was published in the *Western Political Quarterly* in December 1982 (vol. 35, pp. 511-26). Mr. Gross's response follows on pages 648-51 of this issue.

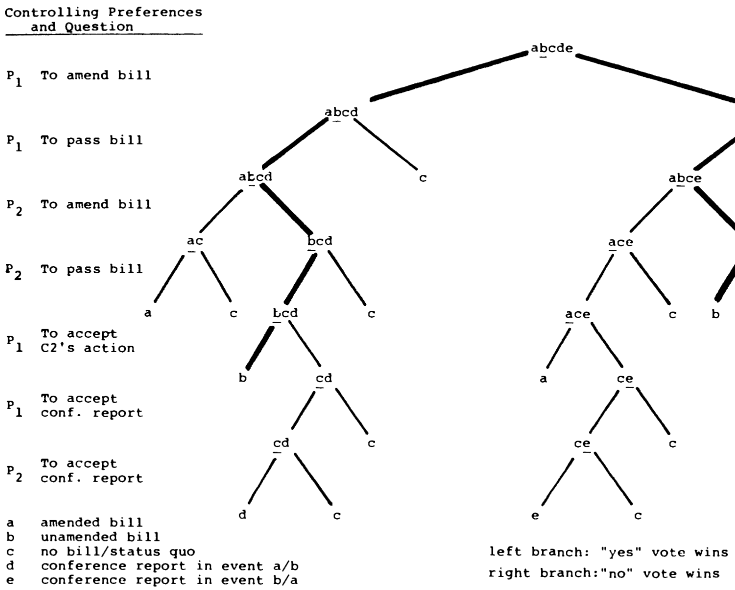

FIGURE 1. A BICAMERAL DIVISION TREE

Given the preferences specified by Gross on p. 519 and applying the "tree method" for analyzing sophisticated voting (called the "multistage method" by McKelvey and Niemi [1978], who provide the definitive characterization), we readily identify b as the sophisticated voting decision. (This is consistent with Gross's verbal conclusion, though it is not entirely clear to me how he reaches it on the basis of his Figure 3.) It is worth noting that this sophisticated voting decision can be reached by two distinct paths down the voting tree (shown in heavy lines). First, Chamber 1 (C1) votes out a (just as it would if it were a unicameral body), then Chamber 2 (C2) votes out b (just as it would if it were a unicameral body), and then C1 votes to accept b (knowing that, if it forces a conference committee, which will report out d, C2 will reject the conference report, bringing about outcome c [no bill] which a majority of C1 regards as worse than b [i.e., b P_1 c]). Alternatively, C1 votes out b in the first place (knowing that b will be the ultimate decision regardless of whether it votes b or a), which C2 then endorses. Note that only the second possibility illustrates Gross's point that a chamber in a bicameral legislature may behave differently from a unicameral body with the same preferences.

Gross's point is more decisively illustrated if we modify his example by giving the conference committee discretion to report either d or e. This can be accomplished by adding another division level to the proper voting tree, giving us the structure displayed in Figure 2. Under this procedure, using the same chamber preferences as before and supposing that the conference committee prefers e to c (i.e., e P_C c), since majorities in both chambers prefer e to c, it turns out that a is the sophisticated decision and that it is reached by a single path: both chambers vote out a and no conference committee is necessary. In this case C2 necessarily behaves differently from the way it would if it were a unicameral legislature with the same preferences. If C2 were to vote out b — as it would as a unicameral (sophisticated or sincere) body — it would force a conference committee, which would report out e (for if the conference reported out d, C2 would reject it, resulting in outcome c, and e P_C c), which would become the decision, and a P_2 e.

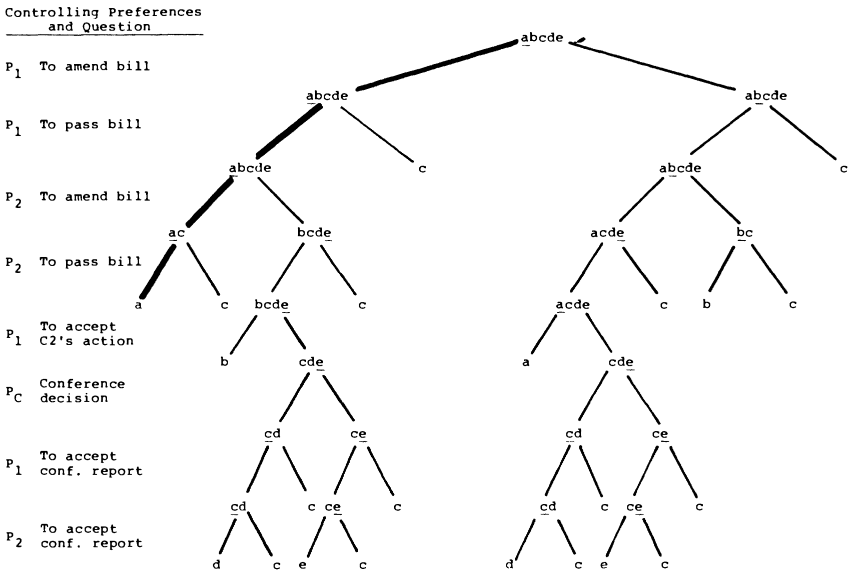

FIGURE 2. A BICAMERAL DIVISION TREE WITH CONFERENCE COMMITTEE DISCRETION

Let us refer to the circumstance in which a single chamber in a bicameral legislature behaves differently from the way a unicameral body with the same preferences would behave (so that, as Gross says, "the analysis of sophisticated voting cannot be undertaken on only one chamber of a bicameral system") as the *bicameral effect*.

Gross attributes the bicameral effect to the fact that "bicameralism results in the generation of alternatives not originally manifested during the initial voting in any one legislative chamber [viz. the possible conference reports," in conjunction with the fact that "the stringent condition of collective rationality" often is not met (1982: 512).

Thus Gross asserts the "generation of additional alternatives" and the absence of "collective rationality" are both necessary for the bicameral effect. But the first part of the assertion is flatly wrong and the second part is at best unclear and probably also wrong.

It is easy to demonstrate that the bicameral effect does not depend on the "generation of additional alternatives." For suppose that all the "second chamber" can do is endorse what the first has done or veto it (in the manner of an Executive with a veto). Then certainly no additional alternatives are generated beyond those considered in the "first chamber." But the bicameral effect may still appear. This is illustrated by the three-alternative example presented in Figure 3. As a unicameral body, C1 would vote out a; as part of bicameral system (or a legislature "checked" by an Executive), it votes out b (for if it voted out a, a would be vetoed and b P_1 c).

Controlling Preferences and Question

P_1 To amend bill

P_1 To pass bill

P_2 To veto bill

P_1	P_2
a	b
b	c
c	a

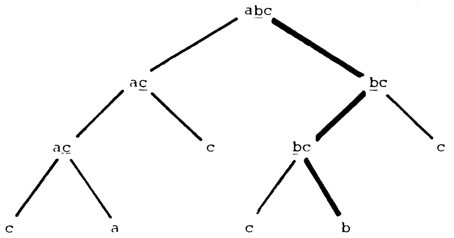

FIGURE 3. SECOND CHAMBER CAN VETO ONLY

Thus, while bicameralism typically may entail the "generation of additional alternatives," this is not necessary for the bicameral effect.

The second part of Gross's assertion — that the absence of "collective rationality" is necessary for the bicameral effect — is not really clear. Sen's Property α refers to choice sets and Gross defines choice sets in terms of some binary social preference relation R (p. 514). What is this R? Social preference in the sense of majority preference? But, as Gross points out elsewhere (pp. 520-21), the meaning of majority preference is ambiguous in the context of bicameralism. Moreover, it is fundamental that majority preference cannot generally rationalize social choice functions even in the absence of bicameralism (or other structural complexities), because of the cycling phenomenon. Finally the connection between "collective rationality" conditions (such as Property α) and the theory of voting even in simple unicameral bodies is unclear because they pertain to two distinct theoretical systems — axiomatic social choice theory and game theory — which are related but by no means isomorphic.

Apparently Gross means to say (I infer this mainly from his discussion on p. 514) that, when additional conference alternatives are generated, the bicameral effect will appear in a given chamber (if and?) only if that chamber would choose differently from out of the broader agenda (including conference alternatives) from the way it would choose out of the narrower agenda (including only alternatives actually voted on in that chamber in the first instance). To formalize a bit, let V_1 designate the set of original (non-conference) alternatives and V_2 the set of additional (conference) alternatives. Then Gross seems to say that the bicameral effect will not appear in a given chamber if $C(V_1 \vee V_2) = C(V_1)$, where $C(V) = \{x \in V{:}xRy \text{ for all } y \in V\}$ and R is the majority preference relation in that chamber. Otherwise the bicameral effect may (or must?) appear in that chamber.

Assuming that this captures what Gross means, I make several observations. First $C(V_1 \wedge V_2) = C(V_1)$ is not Property α; it implies Property α but is not implied by it; Property α requires only that, if $C(V_1 \vee V_2) \wedge V_1 = \phi$, then $C(V_1 \vee V_2) \wedge V_1$ is a subset of $C(V_1)$.[1]

Second, this condition will not be generally applicable unless majority preference in the chamber is acyclic, for a cyclic binary relation cannot generate a choice function over all agendas.

Third and most important, if majority preference is sufficiently transitive that the condition is applicable, its fulfillment does not preclude the bicameral effect. (This is suggested, in fact, by the earlier example of a bicameral effect in the absence of additional conference alternatives, i.e., where $V_2 = \phi$ and $C(V_1 \vee V_2) = C(V_1)$ thus holds trivially.) Consider again the example depicted in Figure 2. For Chamber 1 $C(V_1 \vee V_2) = C(\{a,b,c,d,e\}) = \{a\} = C(\{a,b,c\}) = C(V_1)$; and for Chamber 2, $C(V_1 \vee V_2) = \{b\} = C(V_1)$. Further, if we specify that e P_2 d (Gross does not specify this preference one way or the other), Property α is satisfied in both chambers for all arbitrary agendas. So "collective rationality" holds. But, as we have already seen, the bicameral effect necessarily appears in Chamber 2.

In sum, Gross is quite right in identifying the bicameral effect as a significant feature of bicameral voting processes and one which renders suspect "analyses of sophisticated voting in only one chamber." But his diagnosis of when and why the bicameral effect occurs almost completely misses the mark. It may occur even if no additional non-conference alternatives are generated and even if Property α is met. Rather, the bicameral effect results from the fact that a sophisticated bicameral voting process is characterized not only by strategic interaction within a given chamber (covered by the "conventional" theory of sophisticated voting of Farquharson, etc.) but also by strategic interaction between different chambers as collectivities. A correlative point of some importance is that the bicameral effect does not appear in *sincere* bicameral voting processes (even if additional alternatives are generated and Property α is not met — conditions that make no distinction between sincere and sophisticated voting).

The difference between sincere and sophisticated bicameral voting processes — and thus the source of the bicameral effect in the sophisticated case — can be indicated in the following terms. Suppose we first

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[1] Thus Gross's sentence on the middle of p. 514 beginning "Property α requres . . ." is wrong. Moreover, Property α requires the stated condition to hold for all arbitrarily defined V_1 and V_2, while Gross's condition presumably need hold only for V_1 and V_2 as substantively defined (conference vs. non-conference alternatives).

analyze both sincere and sophisticated voting on a given agenda in a "sovereign" voting body whose decision is final (as in Black, Farquharson, etc.). Now suppose we analyze sincere and sophisticated voting on the same "initial" agenda in a voting body with the same preferences but which is not "sovereign," e.g., one chamber of a bicameral body, a unicameral body "checked" by an Executive with a veto, etc.

Our analysis of sincere voting can proceed in the same fashion in both the sovereign and non-sovereign cases. But our analysis of sophisticated voting must differ in these two cases, reflecting the bicameral effect. Fundamentally, the reason that this is so is that, in the sovereign case, the voting decision of the body is final, whereas, in the non-sovereign case, the voting decision of the body merely sets an agenda for the next stage of the process (except in special cases, as when it defeats a bill). But individual voters in the non-sovereign body have preferences over the possible *final outcomes* of the whole process and *sophisticated* voters in the non-sovereign body choose their voting strategies with an eye to the final decision, not the initial decision of their own chamber.

Perhaps the distinction between sincere and sophisticated voting in the non-sovereign body, and the source of the bicameral effect in the sophisticated case, is made most clearly in methodological form. The voting procedure (bicameral or otherwise) can be represented, as we have seen, by a voting tree. The analysis of *sincere voting proceeds from the top of the tree downward*. The path taken at the first division depends only on the preferences of the relevant voters with respect to the question being voted on and is independent of the structure of the tree branching below; and likewise at subsequent divisions. Thus in general, sincere voting at any earlier stage can be analyzed independently of later stages of the process.

On the other hand, the "tree method" of analyzing *sophisticated voting* — explicitly relied on by Gross —*proceeds from the bottom of the tree upward*. The "anticipated decision" (Miller 1977: 784) or "sophisticated equivalent" (McKelvey and Niemi 1978: 5) at a given division depends critically on the structure of the tree branching downward from the division. It naturally follows that sophisticated voting at earlier stages cannot in general be analyzed independently of the later stages of the process. Finally this argument suggests clearly that if, like Gross, we wish to identify conditions necessary or sufficient to allow "analyses of sophisticated voting in only one chamber of a bicameral legislature," these conditions must refer to the subsequent structure of the voting tree and to the distribution of preferences in both chambers, since these determine the sophisticated equivalents at the divisions representing the voting choice of the chamber in question. And clearly these conditons won't have too much to do with the generation of additional alternatives and won't have anything to do with Property α or any other conditions drawn from axiomatic social choice theory.

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