ELECTION INVERSIONS BY THE U.S. ELECTORAL COLLEGE

Nicholas R. Miller
Department of Political Science
University of Maryland Baltimore County
Baltimore MD 21250 USA
nmiller@umbc.edu

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Abstract

It is well known that the U.S. Electoral College, like other two-tier electoral systems, is subject to *election inversions* such as occurred in the 2000 Presidential election, in which the candidate or party that wins the most votes from the nationwide electorate fails to win the most electoral votes or parliamentary seats and therefore loses the election. In so far as this phenomenon may be 'paradoxical,' it is of a different character from most other voting paradoxes in that it may arise even if there are only two candidates or parties. Moreover, it is straightforward in nature and its occurrence is readily apparent. However, the likelihood of election inversions and the factors that produce them are less apparent, and there has been considerable confusion about the circumstances under which election inversions occur. This paper identifies the sources of election inversions by the Electoral College, establishes logical bounds on the phenomenon, and estimates the frequency and magnitude of inversions on the basis of historical state-by-state Presidential election data.

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An *election inversion* occurs when the candidate (or party) that wins the most votes from the nationwide electorate fails to win the most electoral votes (or parliamentary seats) and therefore loses the election. To describe this phenomenon, public commentary commonly uses such terms as 'reversal of winners,' 'wrong winner,' 'divided verdict,' while the academic literature on voting and social choice uses such terms as 'representative inconsistency,' 'compound majority paradox,' and 'referendum paradox.' Election inversions can occur under U.S. Electoral College or any two-tier electoral system. As is well known, the Electoral College actually produced a 'wrong winner' in the 2000 Presidential election, and it has done so twice before.

In so far as this phenomenon may be 'paradoxical,' it is of a somewhat different character from most other paradoxes in the theory of voting and social choice, in that it may arise even if there are only two candidates. Also unlike many other paradoxes, election inversions are straightforward in nature and occurrences are readily apparent. However, the likelihood of inversions and the factors that produce them are less apparent, and there has been considerable confusion about the circumstances under which election inversions occur. For example, the susceptibility of the Electoral College to inversions is often blamed on the small-state bias in the apportionment of electoral votes and/or the 'non-proportional' or 'winner-take-take-all' manner of casting state electoral votes, but neither of these attributes of the Electoral College is necessary for inversions to occur.

With specific respect to the U.S. Electoral College, I first note the three historical manifestations of election inversions and then discuss one massive but 'latent' inversion in more detail. I then use 'uniform swing analysis' based on historical election data in order to estimate the frequency, magnitude, and direction of potential election inversions. Along the way, I identify three causes of election inversions — 'rounding effects,' 'apportionment effects,' and 'distribution effects' — and examine their separate impacts on the likelihood of election inversions.

1. The Problem of Election Inversions

The President of the United States is elected, not by a direct national popular vote, but by an indirect Electoral College system in which (in almost universal practice since the 1830s) separate state popular votes are aggregated by adding up state electoral votes awarded, on a winner-take-all basis, to the plurality winner in each state. Each state has electoral votes equal in number to its total representation in Congress and since 1964 the District of Columbia has three electoral votes. Therefore the U.S. Electoral College is a two-tier electoral system: individual voters cast votes in the first tier to choose between rival slates of 'Presidential electors' pledged to one or other Presidential candidate, and the winning elector slates then cast blocs of electoral votes for the candidate to whom they are pledged in the second tier. At the present time, there are 538 electoral votes, so 270 are required for election and a 269-269 electoral vote tie is possible.

To the best of my knowledge, the first theoretical work on election inversions was by May (1948), who attempted to calculate the *a priori* frequency of inversions based on a particular probability model of election outcomes. Several years earlier Schattschneider (1942) had noted in passing to the '25%–75% rule' pertaining to inversions that is discussed later. A very insightful geometric analysis of 'Electoral College misrepresentation' has been devised by Sterling (1981), which is fruitfully employed in the latter part of the paper. More recently, Nurmi (1999, 2001, and

2002), Laffond and Laine (2000) and Feix et al. (2004) have addressed the general phenomenon of election inversions in social choice terms, and Chambers (2008) has demonstrated (in effect) that no neutral (between candidates or parties) two-tier electoral rule can satisfy "representative consistency," i.e., preclude election inversions. Empirically based estimates of the expected frequency of Electoral College election inversions have been provided by Merrill (1978) and Ball and Leuthold (1991), and Lahrach and Merlin (2010) have done related work with respect to French local government elections. The fact that Electoral College can produce inversions is regularly invoked by critics of the Electoral College (e.g., Peirce and Longley, 1981; Abbott and Levine, 1991; Longley and Peirce, 1996; Edwards, 2004) and is therefore addressed by its defenders as well (e.g., Best, 1971; Diamond, 1992; Ross, 2004).

'Westminster' single-member-district parliamentary systems (e.g., the U.K., Canada, Australia, India, and New Zealand prior to 1993) are likewise two-tier voting systems, and they produce election inversions at least as frequently as the U.S. Electoral College. Some examples are listed in Table 1. While most of these elections were very close with respects to both votes and seats, the case of Canada in 1979 shows that this is not invariably the case.

These parliamentary systems differ in two important respects from the U.S. Electoral College. First, 'Westminster' systems have *uniform districts* — that is, the districts have equal weight (namely a single parliamentary seat), reflecting (approximately) equal populations and/or numbers of voters. In contrast, Electoral College 'districts' (i.e., states) are (highly) unequal in both population and voters and are likewise unequally weighted, at present ranging from 3 to 55 electoral votes.

Second, the popular vote percentages shown in Table 1 indicate that many of these parliamentary inversions occurred in elections in which third and perhaps additional minor parties received a substantial percent of the vote (and won at least some seats). The presence of third parties typically distorts the relationship between votes and seats for the two major parties. In contrast, Electoral College inversions, like most U.S. elections, have occurred in what were for all practical purposes two-candidate contests. In the following analysis, we will deal entirely with two-party popular vote percentages and, with the exception of a special treatment of the 1860 election, will exclude the few Presidential elections that failed to be 'straight fights,' in that third candidates won some electoral votes.

The U.S. Electoral College has produced the three manifest election inversions listed in Table 2. All were very close with respect to popular votes, and two were very close with respect to electoral votes as well.[1]

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[1] The 1876 election was decided (just before inauguration day) by an Electoral Commission that, by a bare majority and straight party-line vote, awarded all of 20 disputed electoral votes to Hayes. The 1824 election is sometimes also counted as an inversion, in that John Quincy Adams was elected President even though Andrew Jackson had received more popular votes (in the 18 out of 24 states in which presidential electors were popularly elected) than Adams; however, Jackson also won more electoral votes than Adams. But because Jackson did not win the required majority of electoral votes, the election was decided by the House of Representatives, which elected Adams. In 1960, peculiarities with respect to Presidential ballot in Alabama make it unclear exactly how to

In addition to these three historical instances, the Electoral College has produced one massive but "latent" election inversion, which has been recognized as such by Sterling (1981) but by few others. In 1860, the Democratic Party split into Northern and Southern wings, each with its own Presidential candidate. In addition, a fourth candidate, John Bell, was nominated by the remnants of the Whig Party in the South under the label of the Constitutional Union Party. Their popular and electoral vote totals, as shown in Table 3, entail two manifest but inconsequential inversions — namely, Douglas won more popular votes but fewer electoral votes than either Breckinridge and Bell. Under a system of direct popular vote, the two Democratic candidates would have been 'spoilers' against each other, if we suppose that, in the event of the withdrawal of either, the other would have inherited most of his support and therefore would have defeated Lincoln. But under the Electoral College system, Douglas and Breckinridge were *not* spoilers against each other. Let us make the following strong counterfactual suppositions: (1) the Democrats successfully hold their Northern and Southern wings together and thereby win *all* the votes captured by each wing separately, (2) the election is a typical Democratic vs. Republican 'straight fight' and the Democrats also inherit *all* the votes of the Constitutional Union party; and, for good measure, (3) the Democrats win *all* of New Jersey's electoral votes (which, for particular reasons, were split between Lincoln and Douglas). Even so, Lincoln still would have won the 1860 election. The final column of Table 3 shows the results of this counterfactual 1860 election. Of all the states that Lincoln won, he won by less than an absolute majority of the popular vote only in California and Oregon, so the consequence of all these suppositions is that only 11 electoral votes (in California, Oregon, and New Jersey) would switch from the Republican to Democratic column. We will examine this counterfactual two-party variant of the 1860 election in more detail later.

Thus a first cut at estimating the expected frequency of election inversions under the Electoral College — based on the historical record since 1828 (the first election in which almost every state selected presidential electors by popular vote) — is either 3/46 = .06 or (counting the counterfactual 1860) 4/46 = .087. However, with the exception of the counterfactual 1860 election, all inversions occurred in close elections and, considering only elections in which the winner's popular vote margin was no greater than about 3 percentage points, the expected frequency of inversions is considerably higher, namely 3/12 = .25. (See Table 4.) Clearly an important determinant of the probability of an election inversion is the probability of a close division of the popular vote.

2. Popular Votes and Electoral Votes

We now turn to a more informative empirical analysis of election inversions that uses historical state-by-state popular vote percentages to construct what may be called the Popular

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determine the 'popular vote' for President in that state, and thus also nationwide. One (somewhat implausible) reckoning of the Alabama popular vote makes Nixon the national popular vote winner, thereby making 1960 an election inversion. In any event, the 1960 election is excluded from this analysis because a third candidate won electoral votes (see the Appendix).

Vote-Electoral Vote (PVEV) step-function for each historical straight-fight election.[2] The PVEV is based on the kind of 'uniform swing analysis' pioneered by Butler (1951), which has also been called 'hypothetical (single-year) swing analysis' (Niemi and Fett, 1986) and the 'Bischoff method' (see Peirce and Longley, 1981), and which has been employed by Nelson (1974), Garand and Parent (1991), and others in the context of assessing 'partisan bias' in the Electoral College.

The PVEV function is a cumulative distribution function and is therefore (weakly) monotonic. It is a step-function because the 'dependent variable' (EV) is discrete, assuming only whole number values and jumping up in discrete steps as the 'independent variable' (PV) increases (essentially) continuously.

Let us consider the 1988 election as an example. We set up the template used in Figure 1, showing the Democratic popular vote percent on the horizontal axis and the Democratic electoral votes on the vertical axis.[3] The Democratic nominee Michael Dukakis received 46.10% of the two-party national popular vote and won 112 electoral votes (one of which was lost to a 'faithless elector'). This combination of Democratic popular and electoral votes is plotted in Figure 1(a).

Figure 1(b) zooms in on the neighborhood of this plotted point. Of all the states that Dukakis carried, he won Washington (with 10 electoral votes) by the smallest margin of 50.81%, so if the Democratic national popular vote of 46.10% were to decline by 0.81 percentage points (to 45.29%) *uniformly across all states*, Washington would tip into the Republican column and thereby reduce the Democratic electoral vote to 102, as shown in Figure 1(b). In like manner, of all the states that Dukakis failed to carry, he lost Illinois (with 24 electoral votes) by the smallest margin of 48.95%. So if the Democratic national popular vote of 46.10% were to increase by 1.05 percentage points (to 47.15%) uniformly across all states, Illinois would tip Democratic and thereby increase the Democratic electoral vote to 136, as also shown in Figure 1(b).

More generally, we can 'swing' the Democratic vote downwards until the Democratic electoral vote falls to the logical minimum of zero and upwards until it increases to the logical maximum of 538, as is shown in Figure 1(c).[4] This chart displays the PVEV function for 1988, over which Democratic popular support may either rise or fall uniformly across the states and be translated into corresponding Democratic electoral vote totals.

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[2] See the Appendix for details concerning this data.

[3] Remember that, here and elsewhere, popular vote percentages are put on a strictly two-party basis, excluding votes cast for third or other minor candidates, and (with the 1860 exception already noted) we consider only elections in which the two major candidates won all the electoral votes, thus putting everything is put on a strictly two-party basis. We therefore would reach exactly the same conclusions if we organized charts in terms of Republican popular and electoral votes.

[4] Defining the uniform swing in terms of the absolute percent of the total popular vote means that highly lopsided state popular votes in conjunction with very large national swings can create hypothetical party popular vote percentages less than 0% or greater than 100% in some states. But this is of no practical concern because our focus is on hypothetical elections that are close to the 50% mark with respect to the national popular vote.

While the full PVEV function in Figure 1(c) appears to go through the *two-way tie point* corresponding to PV = 50% and EV = 269, a moment's thought suggests that almost certainly it does not go *precisely* through this point. This becomes evident when we zoom in on the center of the chart in Figure 1(d). We see that (i) if Dukakis had won exactly 50% of the popular vote, he would have lost the election with only 252 electoral votes, and (ii) if he had won anything between 50.00% and 50.08% of the popular vote, he still would have lost the election with no more than 260 electoral votes. Thus there is an *inversion interval* 0.08 percentage points wide, within which Dukakis would have won the popular vote but lost the election on the basis of electoral votes. The magnitude of this interval is really more important than the electoral vote split at the 50% popular vote mark, because the likelihood of an election inversion depends on the magnitude of the inversion interval, and the specific number of electoral votes that the 'wrong winner' receives within this interval does not affect who is elected President.

Clearly if the Democratic electoral vote is less than 269 when the Democratic popular vote is exactly 50%, the inversion interval lies above the 50% mark and makes the Republican candidate the 'wrong winner,' and conversely if the Democratic electoral vote is greater than 269. Thus, whether the Democrats get more or less than half the electoral votes when they win 50% of the popular vote determines whether the inversion interval is pro-Democratic (below 50%) or pro-Republican (above 50%, as in 1988). However, the *magnitude* of the Democratic electoral vote deficit or surplus at 50% of the popular vote is unrelated to the *magnitude* of the inversion interval. For example, the Democrats might fall 50 or so electoral votes short of 270 when they win 50% of the popular vote, but California and/or a cluster of smaller states could tip into their column just above the 50% mark giving them 270 or more electoral votes, so the inversion interval could be tiny despite the large electoral vote deficit at the 50% mark.

It should also be noted that, with an even number of electoral votes, there may be a 'tie interval,' within which neither candidate has the required electoral vote majority. Few PVEV functions have tie intervals but those for 1872, 1972, and 2008 do. If a 'tie interval' spans the 50% popular vote mark, there is no inversion interval at all; however no historical PVEV has this attribute.[5]

As is well known, the 2000 election produced an actual election inversion. At first blush, we might think that was because the 2000 PVEV function, shown in Figure 2(a), was quite different from that in 1988 and, in particular, that it entailed a much larger (pro-Republican) inversion interval. While the 2000 interval was larger (but, in absolute terms, only slightly so), the crucial difference between the two elections is the obvious fact that the actual 2000 election itself was *much* closer. The Democratic two-party vote percent popular vote percent was 50.267%, putting it (just) within the 2000 inversion interval, as shown in Figure 2(b). (However, Gore would have won if the inversion interval in 2000 had been as small in 1988.)

[5] The 2008 PVEV chart displayed at the 2010 VPP workshop (based on unofficial preliminary data available about a week after the election) did have such a tie interval. Based on the more recently available official data used here, the 2008 PVEV has a tie interval but it does not span the 50% mark.

For every 'straight fight' Presidential election since 1828, Table 5 shows (a) the actual Democratic popular vote percent, (b) the actual Democratic electoral vote percent, (c) the Democratic electoral vote percent at the 50% popular vote mark, (d) the Democratic popular vote percent required for an electoral vote majority (or, in footnotes, for a tie), (e) the resulting magnitude and direction of the inversion interval, and (f) the absolute magnitude of the interval.[6] Figure 3 graphically depicts the magnitude and direction of the inversion (and tie) intervals in historical sequence (omitting the extreme counterfactual 1860 case).

Several generalizations can be drawn from this data. Setting aside the conspicuous exception of the counterfactual 1860 election, inversion intervals are typically quite small, rarely exceeding two percentage points; the mean of the absolute intervals (i.e., ignoring whether they are pro-Republican or pro-Democratic) is about 0.76 percentage points; and they have exhibited a pro-Republican bias more often than not. However, considering only elections since the mid-twentieth century, the intervals have been smaller, rarely exceeding one percentage point and averaging about 0.5 percentage points, and they exhibit no particular party bias.[7] The 1988 election turns out to have the smallest inversion interval on record.

Based as they are on state-by-state data for all 'straight fight' Presidential elections, these results provide a more informative basis for estimating the likelihood election inversion by the Electoral College. Over all elections, the Democratic two-party popular vote percent is more or less normally distributed with a standard deviation of about 6.2%. Let's set its mean value at 50% (excluding the counterfactual 1860 case, it's actually 49.17%). The mean absolute inversion interval of 0.76% implies a probability of an election inversion of approximately .048. If we consider only 1952 onwards, the popular vote SD increases about 7.0% while the mean inversion interval falls to 0.47%, implying a probability of an election inversion of only about .027. But considering only the six most recent elections, while the average inversion interval falls further to 0.43%, these elections have all been relatively close with a SD of 3.7%, raising the probability of inversions to about 0.046.[8]

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6 Electoral vote percentages are given, since the number of electoral votes varies over time. The counterfactual 1860 is included in the table but excluded from the means in the bottom row.

7 Garand and Parent (1991) employ similar uniform swing analysis for Presidential elections from 1872 through 1984 but, instead of using each PVEV function directly, they use it to estimate the best fitting two-parameter ('representational form' and 'partisan bias') logistic S-curve relating popular and electoral votes to estimate the percent of electoral votes won by the Republican candidate at the 50% popular vote mark, as implied by this best fitting curve. Using such a curve produces quite different and usually much smaller Republican electoral vote percent at the 50% popular vote mark. (Garand and Parent do not report inversion intervals, but the S-curve implies substantially larger intervals as well.) The logistic S-curve estimated on the basis of the PVEV function is much more strongly affected in the vicinity of the 50% popular vote mark by asymmetries in PVEV function resulting from the peculiar politics of the old 'solid South' than the PVEV itself is. (See footnote 9.)

8 It is reasonable to group the 1988-2008 elections together, because they have very similar PVEVs, with pairwise correlations never falling below 0.75 and with the correlations between adjacent elections always at about 0.9 or higher.

The dependence of inversions on the closeness of elections is more clearly displayed in Figure 4. Figure 4(a) shows the distribution of Democratic (tie or) win percentages and the number of resulting inversion intervals over the full historical period, and it makes the historical pro-Republican advantage very evident. Figure 4(b) shows the number of elections with given *absolute* (tie or) inversion intervals; it can reasonably be interpreted as indicating the approximate probability of an election inversion as a function of the popular vote winner's margin above 50% of the two-party vote. If that margin is barely more than 50%, the a priori probability of an inversion is just about 0.5; if it is about 50.5%, Figure 4(b) shows that the probability is about .25; if it is about 51%, the probability is about 0.125, and if it exceeds 52%, the probability is almost zero (in the absence of extreme sectional conflict like 1860). It is worth noting that Merrill (1978) and Ball and Leuthold (1991) come up with quite similar estimates based on entirely different methods.

3. Rounding Effects

The PVEV for 1988 is almost symmetric — that is, if we construct the Republican PVEV and superimpose it on the Democratic one, the two step functions, while distinct in detail, come very close to coinciding, not only in the vicinity of the 50% popular vote mark but over most of its domain, as is shown in Figure 5(a). If the two functions were to coincide precisely, no inversion interval could exist. The small inversion interval results from what we might call the 'rounding error' necessarily entailed by the fact that a PVEV function moves up or down in discrete steps as the popular vote swings up or down. For example, as the Democratic popular vote swings upwards, the pivotal state that gives the Democratic candidate 270 or more electoral votes almost certainly will not tip into the Democratic column *precisely* as the Democratic popular vote crosses the 50% mark but rather a little below or above the 50% mark, so an inversion interval of some magnitude almost always exists. It is also evident that a specific PVEV function allows a 'wrong winner' of one party only, depending on whether the inversion interval lies above or below the 50% Democratic popular vote mark.

However, if we allow small perturbations in the PVEV function, thereby 'thickening' and 'smoothing' it (as suggested in Figure 5(b)), the resulting "generalized" PVEV function passes through the two-way tie point (at PV = 50% and EV = 50%), even though the 'crisp' step function does not. So if the 1988 election had been much closer and the PVEV function had been slightly perturbed, Dukakis as well as Bush could have been a 'wrong winner.'

The PVEV functions for both parties in 1940 and in the counterfactual version of 1860 are displayed in Figures 6(a) and 6(b). These superimposed functions, unlike those for 1988, do not follow each other at all closely, most importantly in the vicinity of the 50% popular vote mark. Thus, in both in 1940 and (especially) in 1860, the 'generalized' PVEV function misses the two-way tie point by a large margin. Inversions are much more likely to occur because they result from, not mere 'rounding effects,' but a fundamental asymmetry in the general character of the PVEV function — in particular, distinct 'partisan bias' in the vicinity of the 50% mark. Even with substantial perturbations of these functions, 'wrong winners' would certainly be Republicans, not Democrats.

The 1940 PVEV exemplifies, and the 1860 PVEV previews in exaggerated form, the substantial pro-Republican bias in historical PVEV functions in the vicinity of the 50% mark that resulted largely from the electoral peculiarities of the old 'Solid South' — in particular, its overwhelmingly Democratic popular vote percentages, combined with its strikingly low voting turnout. While the overall bias was pro-Democratic, in the vicinity of the 50% popular vote mark the bias was pro-Republican. Consider the PVEV for 1940 displayed in Figure 6(a): the Democrats win more electoral votes than the Republicans do for almost all levels of popular vote support, but the Republicans collect more in a narrow range of popular votes in the vicinity of the 50% mark, which of course is precisely the range that matters.[9] The 1860 PVEV provides an even more extreme example.

Bias in the PVEV function can result from either or both of two distinct phenomena, which I call *apportionment effects* and *distribution effects*.[10] The former refers to discrepancies between the popular votes cast within states and the electoral votes cast by states. The latter reflects geographical patterns in the popular vote for the two candidates or parties that makes one candidate's distribution more 'efficient' in winning electoral votes or districts than the other. Either effect by itself can produce election inversions and, in combination, they can either reinforce or counterbalance each other.

4. Apportionment Effects

In order to assess apportionment effects, we start with the benchmark of a *perfectly apportioned* two-tier electoral system, in which apportionment effects are eliminated because electoral votes are apportioned among the states in a way that is precisely proportional to the total popular vote cast within each state. It follows that, in a perfectly apportioned system, a candidate who wins X% of the electoral vote carries states that collectively cast X% of the total popular vote.[11] This concept is introduced as an analytical tool; as a practical matter, an electoral system can be perfectly apportioned only retroactively — that is, after the popular vote in each state is cast and counted.[12]

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9 I believe that this consideration largely determines the Garrand and Parent (1991) finding, based on smooth S-curves estimated on the basis of the entire PVEV, that the Electoral College has historically had a pro-Democratic bias.

10 In other contexts (e.g., Grofman et al. 1997), it is useful to separate out a third effect — namely, *turnout effects* — that here is subsumed under apportionment effects.

11 Note that this says nothing about the popular vote margin by which the candidate wins or loses states and, in particular, it does say or imply that the candidate wins X% of the national popular vote,

12 However, Alan Natapoff (1996) has proposed that each state's electoral vote should be made (retroactively) proportional to its share of the national popular vote, while retaining the within-state winner-take-all feature. At first blush, such a system seems to create seemingly perverse turnout incentives in a 'non-battleground' states, as supporters of the candidate who is destined to lose the state can help their preferred candidate by abstaining, thereby reducing the number of electoral votes collected by the statewide winner. However, Natapoff views this as a positive aspect of his proposal, because it gives otherwise impotent voters in 'non-battleground' states some influence on the national outcome, in that supporters of the statewide loser can help their candidate nationally by abstaining and supporters of the statewide winner can help their candidate by voting. Moreover, the prospective statewide winner has an incentive to moderate his stands or otherwise make concessions to his opponent's supporters in order to induce them to vote rather than abstain.

Apportionment effects encompass whatever causes deviations from perfect apportionment. The U.S. Electoral College system is imperfectly apportioned, for at least six reasons.

(1) House seats (and therefore electoral votes) must be apportioned in (relatively small) *whole numbers*, and therefore cannot be *precisely* proportional to *anything*.

(2) There are many *different methods* of apportioning whole numbers of seats on the basis of population, none of which is uniquely best (Balinski and Young, 1982).

(3) House (and therefore electoral vote) apportionments are anywhere from *two to ten years out-of-date* at the time of a Presidential election.

(4) The apportionment of electoral votes is *skewed in favor of smaller states*, as they are guaranteed a minimum of three electoral votes (due to their guaranteed one House seat and two Senate seats) and (approximate) proportionality begins only after that.

(5) The size of the House is not fixed by the Constitution and can be changed by law (as it was regularly in the nineteenth century), so the magnitude of the small-state bias can be reduced (or enhanced) by law, by increasing (or reducing) the size of the House.[13]

(6) House seats (and therefore electoral votes) are apportioned to states on the basis of their *total population* and not on the basis of their (i) voting age population, or (ii) voting eligible population (excluding non-citizens, etc.), or (iii) number of registered voters, or (iv) number of actual voters in a given election.[14]

Similar apportionment imperfections apply (in greater or lesser degree) in all two-tier electoral systems.

While imperfect apportionment may create bias in the PVEV function, it need not do so. Actual bias depends on the extent to which states' advantages or disadvantages with respect to apportionment effects are correlated with their support for one or other candidate or party. The maximum bias that can arise from imperfect apportionment can be measured by ranking the states by their degree of advantage with respect to actual apportionment relative to perfect apportionment and cumulating both electoral votes and total popular vote shares over this ranking until the required majority of electoral votes is reached and noting the corresponding share of the national popular vote that has been accounted for. Today this popular vote share runs about 45% but it was considerably larger in the past. Table 6 shows this percentage for all elections and also identifies the states

[13] See Neubauer and Zeitlin (2003) for an analysis of how changes in House size would have affected the 2000 Presidential election.

[14] In addition, until slavery was abolished by the Thirteen Amendment in 1865, House seats were apportioned on the basis of the total free population plus three fifths of 'all other persons' (who certainly did not vote). In several elections prior to the ratification of the Nineteenth Amendment women could vote in some states but not others. Most notably, in 1916 Illinois had only 29 electoral votes compared with New York's 45, but the total popular vote in Illinois (with women's suffrage)considerably exceeded that in New York (without women's suffrage).

receiving the maximum bonus and penalty from the prevailing imperfect apportionment. (The two remaining columns in Table 6 will be discussed later.)

While perfect apportionment is not feasible in practice, it would be entirely practicable to base the apportionment of electoral votes on House seats only and thereby remove the small-state advantage. Table 7 shows summary data for all elections with respect to this revised apportionment. One might expect that 'improving' apportionment in this way would reduce the frequency of election inversions. Indeed, with electoral votes apportioned in this manner, Gore would have won the 2000 election by winning 225 electoral votes out of 436 and Tilden would have won the 1876 election by winning 150 electoral votes out of 293. However, Cleveland would still have decisively lost to Harrison in 1888 and Wilson would have lost the 1916 election with only 216 electoral votes out 435, despite a modest majority of the popular vote. So with respect to actual election inversions, the modified apportionment would eliminate two but not all three instances and it would create one new instance. While House apportionment retains the overall Republican bias of the existing apportionment, it does on average slightly reduce the magnitude of the bias. At the same time, it slightly increases the average magnitude of absolute inversion intervals. One 'spanning' tie interval shows up in this data.

Let us now examine the impact of apportionment effects in the counterfactual 1860 election, which was based on especially imperfect apportionment.

(1) The southern states (in most cases for the last time[15]) benefitted from the three-fifth compromise giving them partial credit for their non-voting slave populations.

(2) Southern states had on average smaller populations than northern states and therefore benefitted disproportionately from the small-state advantage in apportionment.

(3) Even within the free population, suffrage was more typically restricted in the South than in the North (where close to universal adult male suffrage prevailed).

(4) Turnout among eligible voters was generally lower in the South than the North.

But *all* of these apportionment effects favored the South and therefore the Democrats. Thus the massive pro-Republican election inversion was entirely due to distribution effects and the magnitude of the inversion interval would have been even greater in the absence of the counterbalancing apportionment effects.

While perfect apportionment is not feasible in practice, we can apply it analytically and retroactively. Figure 7(a) compares the 1988 PVEV functions and inversion intervals under both perfect and actual apportionment. Clearly apportionment effects were very small in this election. Figures 7(b) and 7(c) make the same comparisons for 1940 and 1860. Here apportionment effects are very substantial at low Democratic popular vote percentages and remain quite substantial around the 50% mark in 1940 and around the 60% mark in 1860. The 1940 chart reflects the 'Solid South' effect rooted in the 1860 election.

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[15] The few slave states that did not secede from the union retained this apportionment advantage in the 1864 election. By the time of the 1868 election, there were no 'other persons.'

Table 8 shows summary data for all elections with respect to perfect apportionment. We might expect that 'perfecting' apportionment would greatly reduce the frequency and magnitude of election inversions. With respect to actual election results, perfect apportionment produces exactly the same results as House apportionment by 'correcting' the same two historical inversions, failing to 'correct' the third, and creating the same new inversion. Perhaps surprisingly, perfect apportionment actually increases the degree of Republican bias and (as a consequence of this) considerably increases the average magnitude of absolute inversion intervals.

5. Distribution Effects

Distribution effects in two-tier electoral systems result from the winner-take-all feature at the state (or district) level. Distribution effects can be powerful even with uniform districts and/or perfect apportionment. If one candidate's (or party's) popular vote is more 'efficiently' distributed over states (or districts) than the other's, an election inversion can occur even with perfect apportionment.

The simplest possible example of distribution effects producing an election inversion in a small, uniform, and perfectly apportioned district system is provided by nine voters in three districts. Suppose that the individual votes for candidates D and R in each district are as follows: (R,R,D) (R,R,D) (D,D,D). Thus the election outcome is as follows:

	Popular Votes	*Electoral Votes*
D	5	1
R	4	2

Since R's votes are more 'efficiently' distributed than D's (whose support is 'wastefully' concentrated in the third district), R wins a majority of districts with a minority of popular votes.

More generally, suppose we have k uniform districts each with n voters. To avoid the problem of ties, let us assume both k and n are odd numbers. A candidate can win by carrying a bare majority of $(k + 1) / 2$ districts each with a bare majority of $(n + 1) / 2$ votes. Thus a candidate can win with as few as $[(k + 1) / 2] \times [(n + 1) / 2] = (n\,k + n + k + 1) / 4$ efficiently distributed total votes. With $n = 3$ and $k = 3$, the last expression is 4/9 = 44.4%, but as n and/or k become large, the last expression approaches a limit of $n\,k / 4$, i.e., 25% of the total popular vote.

Stated more intuitively, if the number of districts is fairly large and the number of voters is very large, the most extreme logically possible election inversion in a perfectly apportioned system results when one candidate or party wins just over 50% of the popular votes in just over 50% of the uniform districts or in non-uniform states that collectively have (just over) 50% of the electoral votes. These districts also have (just over) 50% of the popular vote (because apportionment is perfect). The winning candidate or party therefore wins just over 50% of the electoral votes with just over 25% of the popular vote and the other candidate, though winning almost 75% of the popular vote, loses the election, producing a massive election inversion. In the resulting PVEV, the inversion interval is (just short of) 25 percentage points wide. (If the candidate or party with the favorable vote distribution is also favored by imperfect apportionment, the inversion interval could be even greater.) This '25% - 75%' rule pertaining to distribution effects has been noted in passing by Schattschneider

(1942, p. 70) and more formally by May (1948), Laffond and Laine (2000), and most likely others.

In the counterfactual 1860 Lincoln vs. anti-Lincoln scenario (unlike any actual election), the popular vote distribution over the states approaches the logically extreme 25%-75% pattern. In the counterfactual election, Lincoln carries all the northern (free) states except New Jersey, California, and Oregon, generally by modest popular vote margins that never exceed about 60%. These states hold somewhat more than half the electoral votes and a larger majority of the (free) population. The anti-Lincoln opposition carries all the slave states by essentially 100% margins. (No Lincoln-pledged electors ran in any of the states that would subsequently secede from the Union.) The opposition also carries California and Oregon by substantial margins and New Jersey by a narrow margin. Altogether these states hold somewhat less than half of the electoral votes and substantially less than half of the (free) population.

Sterling (1981) has devised a geometric representation that allows us to visualize apportionment effects and distribution effects together, and which can be used to display the extent to which the two effects favor one party or the other. A Sterling diagram is essentially a histogram that shows the popular vote for each state, ranked in order from the strongest to weakest for the winning party, with the width of each 'bar' proportional to the total popular vote cast in that state.

Figure 8(a) shows the Sterling diagram for the 1988 election. Selected 'bars' are explicitly drawn in and labeled by state. Running from the most Republican state of Utah to the least Republican 'state' of the District of Columbia, it is Michigan, beating out Colorado by about 0.03% (not enough to show up in the chart) that tips the Republican electoral vote over 270. Once Michigan is in their column, the Republicans are carrying states with 49.43% of the national popular vote, as indicated by the vertical dashed line. The fact that this falls below the 50% mark reflects the (very small and previously noted) apportionment effect favoring the Republicans in 1988.[16] The area of the whole rectangle making up the Sterling diagram represents all 100% total national (two-party) popular vote. The shaded area below the tops of the bars represents the 53.9% of the popular vote won by the Republicans and the unshaded area above the top of the bars represents the 46.1% of the popular votes won by the anti-Lincoln opposition.

The next-to-last column in Table 6 shows the total popular vote cast by the minimum number of states, ranked in descending order of Democratic strength, required for a Democratic electoral vote majority. For 1988, this figure is 54.58%. At first blush, one might expect that this figure would be 50.57% since, as we have seen, the corresponding Republican figure is 49.43%. However, the Republican figure includes the percent of the national popular vote (4.01%) cast in the pivotal state of Michigan, so the Democrats needed to carry states casting 50.57% + 4.01% = 54.58% of the national popular vote to win a majority of electoral votes. To take a less hypothetical example, Table

[16] Note that the interval between this dashed vertical line and the 50% mark is not the same as the inversion interval, as it pertains only to apportionment. The inversion interval is the absolute difference between 50% and the smallest national popular vote percent for a candidate that produces an electoral vote majority. This interval is the absolute difference between 50% and the share of the popular vote cast by the smallest set of states (ranked by party strength) that produces an electoral vote majority. In the absence of apportionment effects, this interval must be zero.

6 shows that to win in 2000 Gore needed to carry states (Florida plus all more Democratic states) casting 56.84% of the national popular votes. Thus, apart from Florida, Bush carried states (all more Republican than Florida) casting 43.16% percent of the popular vote, but to win the election he needed these states plus Florida (which cast 5.74% of the national popular vote), for a total 43.16% + 5.74% = 48.90% of the national popular vote. (The prospective electoral vote splits were likewise 'unbalanced'. By winning Florida and the election, Bush won 246 + 25 = 271 electoral votes; if Gore had won the Florida and the election, he would have won 267 + 25 = 292 electoral votes.) More generally, the reason that the figures in the next-to-last column in Table 5 bounce around so much is because pivotal states are typically big states, and tiny shifts in the national popular vote split between the two candidates can shift pivotal state one way or another and thus have a big impact on the percent of the national popular vote cast by states carried by one or the other candidate.

Figure 8(b) is a standard Sterling diagram that demarcates different portions of the total Republican popular vote. The dark shaded portion represents the portion of the total popular vote essential for 270 electoral votes, and it is very close to that given by the '25% – 75% rule.' However, because apportionment effects work slightly in favor of the Republicans, it is actually slightly less, rather than slightly more, than 25% of the total. The lightly shaded portion represents 'surplus' Republican popular votes divided into three categories: (i) 'surplus' votes (in excess of 50%) in the states essential for 270 electoral votes, (ii) all votes up to 50% in 'surplus' states (in excess of 270 electoral votes), and (iii) votes that are 'surplus' in both respects.

We can get a direct indication of distribution effects in this or any other election comparing the percent of all votes that are 'surplus' (or 'wasted') to one or other party and thereby determine which party has the most efficient distribution of votes. In Figure 8(b), it appears that the Republicans have more "surplus" votes than the Democrats, but this comes about because the Republicans have a (very small) advantage due to apportionment effects and, of course, because also they won more popular (and electoral) votes than the Democrats. We can modify a Sterling diagram by adjusting the diagram in two ways to remove these factors. First, we reallocate electoral votes among the states so that they are perfectly apportioned. Thus the horizontal axis now shows both the percent of the popular vote cast in, and of the electoral vote cast by, the states as ranked, so the dashed vertical lines at the 49.43% mark that gives the Republicans an electoral vote majority shifts (slightly) upwards to (just above) the 50% mark. (Clearly this adjustment makes very little difference in 1988.) Second, we 'swing' the Republican vote uniformly downward (by simply shifting the tops of the state bars uniformly downward) until the election is a *perfect tie*, in the specific sense that the popular vote is tied in the pivotal state (Michigan) that produces 270 electoral votes, so that the median or pivotal (state) bar has a height of 50%.[17]

Figure 8(c) shows the Sterling diagram for 1988 with these two adjustments. Clearly in such

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[17] In this sense, the 2000 election was only 537 votes away from a perfect tie. Note that a perfect tie is almost certainly not a 'two-way tie,' since almost certainly the national popular vote is not tied (as 2000 also illustrates). While the popular vote is likely to be very close, the only logical constraint remains that given by the 25% – 75% rule.

a diagram, (just under) 50% of the total popular votes (in the upper-left and lower-right quadrants) are 'surplus' to one other party. The adjusted diagram shows how this fixed proportion of surplus votes is divided between the two parties. In the absence of distribution effects (and in a perfect tie election with no apportionment effects), surplus votes would be equally divided between the two parties (25% for each). In 1988 the surplus votes were almost equally divided in this manner: 25.24% for Republicans and 24.76% for Democrats.

The 1988 election provides an example of an election with very small apportionment and distribution effects. The counterfactual 1860 election, in contrast, provides by far the most spectacular example of large apportionment and distribution effects and, in particular, of distribution effects so highly in favor of the Republicans that they totally overwhelm the somewhat more modestly pro-Democratic apportionment effects. Figure 9(a) shows the standard Sterling diagram for 1860. Due to the Democratic apportionment advantage, the Republicans had to carry states casting almost 60% of the popular vote to win, and they actually carried states casting about 67% of the popular vote. Figure 9(b) shows the Sterling diagram for 1860 adjusted for perfect apportionment and a uniform swing against the Republicans sufficient to bring about a perfect tie election (ignoring the fact that such a swing makes the Republicans popular vote negative in Southern states in which they already won zero votes). It thereby isolates distribution effects and shows the massive Republican advantage in this respect: the Democrats 'wasted' twice as many votes as the Republicans did. The final column in Table 6 shows the Democratic share of 'surplus' in all elections.

6. Conclusions

In the absence of both apportionment effects and distribution effects, the PVEV function for an election would indicate that the percent of the popular vote that gives the Democrats an electoral vote majority is 50% plus or minus a small amount due to rounding effects. Since these rounding effects are small, they may be ignored to good approximation, so we would expect the inversion interval to be zero under these circumstances.

For each election, we have calculated the minimum percent of the popular vote that gives the Democrats an electoral vote victory, as reported in Table 5. Call this quantity D_{ad}, as it reflects both apportionment effects and distribution effects; thus $(50\% - D_{ad})$ is the (total) Democratic inversion interval. While we have examined distribution effects in terms of the balance of surplus votes for each party as displayed in modified Sterling diagrams, this does not translate into a direct estimate of the impact of distribution effects on the magnitude of the inversion interval. But in fact we already know this quantity, since we have calculated the minimum percent of the popular vote that would give the Democrats an electoral vote majority in the event that apportionment effects did not work to the net advantage of either party, since this is equivalent to the percent under perfect apportionment, as reported in Table 8. Call this quantity D_d, since it reflects distribution effects only; thus $(50\% - D_d)$ is the Democratic inversion interval due to distribution effects only. What we have not yet calculated is the minimum percent of the popular vote that would give the Democrats an electoral vote majority in the event that distribution effects did not work to the net advantage of either party. Call this quantity D_a, since it reflects apportionment effects only; thus $(50\% - D_a)$ is the Democratic inversion interval due to apportionment effects only. However, the magnitude of D_a can be inferred on the basis of the following accounting identity:

$$D_{ad} \;=\; 50\% + (50\% - D_d) + (50\% - D_a).$$

Thus

$$D_a \;=\; 50\% + 50\% - D_d + 50\% - D_{ad} \;=\; 150\% - (\,D_d + D_{ad}).$$

For example, for 1988

$$D_a = 150\% - (50.05 + 50.08) = 49.87\%,$$

giving the accounting identity

$$50.08\% \;= 50\% + 0.13\% - 0.05\%.$$

Table 9 shows this accounting identity for every election and thereby indicates the magnitude and direction of apportionment and distribution effects in every election and it therefore summarizes the main results of this paper.

Further research along these lines can proceed in a number of directions, many of which I am already pursuing. These include the following.

The non-straight-fight elections that, with the exception of 1860, were eliminated from consideration here could be examined, in the manner of 1860, in terms of various two party scenarios, by combining the votes for candidates in various ways to produce two-party contests. More generally, the way in which multi-candidate or multi-party elections (such as are increasingly found in 'Westminster' parliamentary systems) affect the likelihood of election inversions could be studied directly. However, this would require fundamentally different and more complicated analytical methods.

A number of 'reforms' of the U.S. Electoral College (apart from its total abolition and replacement by a one-tier direct election system) have been proposed over the years, including several 'district' and 'proportional' plans. It remains to be determined whether these reforms would make election inversions more or less likely.

The notion of perturbations in a PVEV function, which was treated informally here, can be treated more formally by simulating elections on the basis of given PVEV with random fluctuations (i.e., non-uniform swings) at the state, regional, or national level.

Finally, a theoretically productive approach would be to estimate the probability of election inversions in random or 'Bernoulli' elections, in which voters decide how to vote by independently flipping fair coins. Such elections can be straightforwardly simulated, and this is the same probability model that provides a practical interpretation of the 'absolute Banzhaf voting power measure,' i.e., the probability of casting a decisive vote in such an election, and voting power analysis has well-known applications to two-tier electoral system such as the Electoral College. Indeed, Feix et al. (2004) have already estimated, by means of simulations, the probability of election inversions in uniform and perfectly apportioned two-tier electoral systems. As the number of districts increases, this probability turns out to quickly approach a limit of about .205. My own preliminary work along the same lines indicates that the probability of inversions is somewhat greater than .205 in Electoral College simulations. To what extent this is due to non-uniform districts or to imperfect apportionment is as yet unclear. One advantage of the random election approach is that no systematic distribution effects will appear and estimates of inversion probabilities, which therefore will reflect only the properties of the electoral institutions themselves, not of more contingent features pertaining to the geographical basis of party support in any particular historical period.

Appendix: Presidential Election Data

The 1828-2000 Presidential election data used here comes from Congressional Quarterly's *Guide to U.S. Elections*, which is based on the Interuniversity Consortium for Political and Social Research (ICPSR) Historical Election Returns file. See the p. xiv in the *Guide* for further details. The 2004 and 2008 data comes from David Leip's *Atlas of U.S. Elections* at **http://uselectionatlas.org/**, which is based on information from state election agencies. For present analytic purposes, it was necessary or expedient to make the following adjustments in the data.

1. All state and national popular vote percentages are based on the major two-party vote only, excluding popular votes cast for third-party and other minor Presidential candidates.

2. Apart from 1860 (for which we consider the 'Republican vs. anti-Republican' counterfactual two-party variant for its extreme characteristics), the following elections are set aside because third-party candidates won electoral votes by carrying at least one state:

   1832 Wirt (Anti-Masonic Party) won 8 electoral votes;
   1856 Fillmore (American Party) won 8 electoral votes;
   1860 Brekinridge (Southern Democrat) won 72 electoral votes and Bell (Constitutional Union Party) won 39 electoral votes;
   1892 Weaver (Populist Party) won 22 electoral votes;
   1912 T. Roosevelt (Progressive Party) won 88 electoral votes;
   1924 LaFollette (Progressive Party) won 13 electoral votes;
   1948 Thurmond (Southern Democrat) won 38 electoral votes;
   1960 Byrd (Southern Democrat) won 14 electoral votes (cast by 'unpledged' electors);
   1968 Wallace (American Independent Party) won 45 electoral votes.

3. Despite significant third-candidate popular votes in 1980 (Anderson), 1992 (Perot), and 1996 (Perot), these elections are not excluded because Anderson and Perot carried no states and therefore won no electoral votes. The popular votes for Anderson and Perot are excluded from popular vote totals (like popular votes for minor candidates in all elections).

4. Because of the general-ticket system for electing party-pledged electors, each state's electoral vote is normally undivided. However, divisions in state electoral votes occur in three circumstances:

   (a) when a 'faithless' elector violates his or her pledge and casts a 'protest' electoral vote for another candidate, which occurred in 1948, 1956, 1960, 1968, 1972, 1976, 1988, 2000, and 2004);

   (b) when electors are elected from districts rather than statewide, and each major-party candidate carries at least one district, as happened in Michigan in 1892 and Nebraska in 2008; and

(c) when electors are elected at-large but individually rather than on a general ticket, as happened with some frequency in the 19th century, in California and North Dakota in 1912, and in Alabama in 1960.

Consistent with the almost universal practice and present analytical purposes, all calculations assume that states cast undivided electoral votes for the state popular vote winner. (Thus, McCain in 2008 is credited with all five electoral votes from Nebraska and Gore in 2000 is credited with 267 electoral votes). When electors are elected at-large but not on a general ticket system, standard records of the Presidential vote by state (including those relied on here) credit each Presidential candidate with the popular vote for his party's leading elector.

5. The South Carolina legislature appointed presidential electors through 1860. These electors were always Democrats, but in 1832 and 1836 they cast their electoral votes for an "Independent Democrat" rather than the national Democratic party nominee. The Delaware legislature appointed electors in 1828 (pledged to J. Q. Adams), the Florida legislature appointed electors in 1868 (pledged to Seymour), and the Colorado legislature appointed electors in 1876 (pledged to Hayes).

   In the calculations pertaining to the actual Electoral College and its variant based on "House" electoral votes only, South Carolina is counted as voting 100% Democratic but casting 0% of the national popular vote, and Delaware, Florida and Colorado are treated in parallel manner. For purposes of making perfect apportionment calculations for the latter three states, I use the total popular vote for governor in the same year (or in 1829 in the case of Delaware) to take the place of the (non-existent) total vote for Presidential electors. This data came from Table x.x, p. 360, of Walter Dean Burnham's *Voting in American Elections*. However, in South Carolina, the legislature appointed the governor as well as Presidential electors. Therefore I use the total vote for U.S. House candidates to take the place of the Presidential vote. This was calculated using Table 2.2 (Potential Electorate Estimates), pp. 115-119, together with Table 8.3b (Estimated House Turnout), pp. 401-410, in Burnham's book.

6. In 1832, Whig presidential electors were pledged to different candidates in different states. In these calculations the popular and electoral votes for the Whig candidates are simply added up to get a national Whig popular vote percent and electoral vote total, so the calculations treat 1832 as a normal two-party election.

7. In 1860, Democratic 'fusion' (i.e., anti-Lincoln) elector slates that included both some prospective electors pledged to Douglas and others to Breckinridge (and in at least one case several pledged to Bell), were run in a number of Northern states, sometimes in competition with "pure" Douglas slates (see Fite, 1911, p. 223). None of the 'fusion' slates won, but they make apportioning popular vote support between Douglas and Breckinridge a somewhat arbitrary matter. Since we use only the counterfactual version of 1860 in which Lincoln runs against a unified opposition, we can sidestep these complexities.

8. In 1872, the Democratic (and 'Liberal Republican') candidate Greeley died after the Presidential election but before the casting of electoral votes. Three Democratic electors in Georgia cast their electoral votes for their deceased nominee, while the other Democratic

electors scattered their votes among four living candidates. Congress refused to count the three Greeley electoral votes from Georgia, and it also refused to count electoral votes (cast for Grant) from Arkansas and Louisiana, due to disruptive conditions in those states. The scattered Democratic electoral votes (including the three rejected votes for Greeley) are counted toward the Democratic total and the Arkansas and Louisiana popular and rejected electoral votes are counted toward the Republican total, so the calculations treat 1872 as a normal national election (apart from the absence of Mississippi, Texas, and Virginia, which had not yet been readmitted to the union).

Table 1. Election Inversions in 'Westminster' Parliamentary Systems

Country	*Election*	*Leading Parties*	*Pop. Vote %*	*Seats*
Britain	1929	Conservatives	38.1	260
		Labour	37.1	287
Britain	1951	Labour	48.78	297
		Conservatives	47.97	302
Britain	1974 (Feb.)	Conservatives	37.9	297
		Labour	37.2	301
New Zealand	1978	Labour	40.4	40
		National	39.8	51
New Zealand	1981	Labour	39.0	43
		National	38.8	47
Canada	1979	Liberals	40.1	114
		Conservatives	35.1	136

Table 2. The Three Historical Election Inversions by the U.S. Electoral College

Election	*EC Winner [EV]*	*EC Loser [EV]*	*EC Loser's 2-P PV%* *
1876	Hayes (R) [185]	Tilden (D) [184]	51.53%**
1888	Harrison (R) [233]	Cleveland (D) [168]	50.41%
2000	Bush (R) [271]	Gore (D) [267***]	50.27%

* EC loser's two-party popular vote percent.

** Unlike Gore and Cleveland, Tilden won an absolute majority (50.97%) of the total popular vote (for all parties/candidates).

*** Gore lost one electoral vote to a "faithless elector."

Table 3. The 1860 Election: A Latent But Massive Inversion

Candidate	*Party*	*Pop. Vote*	*EV*	*Unified Dem*	*Unified Opp.*	*EV*
Lincoln	Republican	39.82%	180	39.82%	39.82%	169
Douglas	Northern Democrat	29.46%	12	47.55%		
Breckinridge	Southern Democrat	18.09%	72		60.16%	134
Bell	Constitutional Union	12.61%	39	12.61%		

Table 4. The Twelve Closest Presidential Elections

Election	*Winner's Popular Vote Margin**	*Winner's Electoral Vote Percent*
1836	+1.74%	57.82%
1844	+1.46%	61.82%
1876	−3.02%	50.14%
1880	+0.02%	57.99%
1884	+0.25%	54.61%
1888	−0.80%	58.10%
1916	+3.13%	52.17%
1960	+0.17%	56.74%
1968	+0.71%	55.95%
1976	+2.10%	55.20%
2000	−0.51%	50.37%
2004	+2.46%	53.16%

*Winner's minus loser's popular vote percent (based on total popular vote)

Table 5. Electoral College Inversion Intervals

ELECTION	DEM 2-PARTY VOTE PERCENT	DEM ELECTORAL VOTE PERCENT	DEM ELECTORAL VOTE PERCENT AT PV = 50%	DEM ELECTORAL VOTE WIN PERCENT	DEM EV INVERSION INTERVAL	ABSOLUTE EV INVERSION INTERVAL
1828	56.19	72.03	44.83	50.65	-.65	.65
1836	50.87	57.82	58.84	49.68	.32	.32
1840	47.52	20.41	52.04	49.32	.68	.68
1844	50.74	61.82	48.73	50.20	-.20	.20
1848	47.33	43.79	66.90	49.22	.78	.78
1852	53.69	85.81	39.53	50.95	-.95	.95
1860	60.17	44.22	41.91	61.26	-11.26	11.26
1864	45.99	8.97	45.73	50.41	-.41	.41
1868	47.34	27.21	47.96	50.75	-.75	.75
1872	44.07	18.03	58.74	48.26 [1]	1.74	1.74
1876*	51.53	49.86	44.17	51.78	-1.78	1.78
1880	49.99	42.28	42.28	50.95	-.95	.95
1884	50.13	54.61	45.64	50.09	-.09	.09
1888*	50.41	41.90	35.91	50.97	-.97	.97
1896	47.81	38.93	48.10	50.23	-.23	.23
1900	46.83	34.68	42.28	51.14	-1.14	1.14
1904	39.99	27.94	50.42	49.83	.17	.17
1908	45.50	32.30	46.79	50.81	-.81	.81
1916	51.64	51.98	47.83	51.46	-1.46	1.46
1920	36.17	23.92	46.70	51.81	-1.81	1.81
1928	41.20	16.38	56.12	48.63	1.37	1.37
1932	59.16	88.89	48.78	50.09	-.09	.09
1936	62.45	98.49	42.56	51.68	-1.68	1.68
1940	54.97	84.56	40.11	51.51	-1.51	1.51
1944	53.78	81.36	39.74	51.26	-1.26	1.26
1952	44.60	16.76	46.33	50.36	-.36	.36
1956	42.25	13.94	51.22	49.67	.33	.33
1964	61.34	90.33	55.39	48.97	1.03	1.03
1972	38.21	3.16	50.74	49.70 [2]	.30	.30
1976	51.05	55.20	47.21	50.19	-.19	.19
1984	40.80	2.42	43.12	50.83	-.83	.83
1988	46.10	20.82	46.84	50.08	-.08	.08
1992	53.46	68.77	48.88	50.65	-.65	.65
1996	54.74	70.45	51.86	49.58	.42	.42
2000*	50.23	49.63	44.05	50.27	-.27	.27
2004	48.76	46.84	52.79	49.82	.18	.18
2008	53.69	67.66	51.67	49.14 [3]	.86	.86
MEAN**	49.18	46.39	48.08	50.30	-.30	.76

* Actual election inversion
** Excluding 1860

1 Tie interval beginning at 47.64%
2 Tie interval beginning at 49.24%
3 Tie interval beginning at 48.84%

Table 6. Maximum Apportionment Effects and Surplus Votes

ELECTION	MIN STATE PV PC FOR EV MAJ	(1) MAX APPORT BONUS	ST	(1) MAX APPORT PENALTY	ST	DEM STATE PV PC FOR EV MAJ	DEM **SURPLUS** VOTE PC
1828	30.32	-15.29	VA	24.66	NY	41.72	27.32
1836	36.02	-12.63	VA	18.00	OH	53.36	23.01
1840	34.79	-12.56	VA	11.88	OH	52.52	22.51
1844	37.05	-7.14	VA	12.53	NY	61.84	19.29
1848	40.01	-7.50	SC	13.95	PA	59.48	19.48
1852	35.43	-5.86	SC	13.77	NY	58.56	21.65
1860	46.05	-8.00	SC	10.85	IL	46.05	34.48
1864	40.36	-5.65	KY	9.42	NY	58.18	20.30
1868	35.57	-5.77	TN	10.47	NY	54.59	23.82
1872	44.48	-3.10	GA	12.11	NY	53.57	21.38
1876	41.33	-3.02	GA	9.68	NY	51.56	25.82
1880	40.28	-4.46	GA	10.24	NY	52.54	24.68
1884	39.80	-6.09	GA	10.40	NY	51.13	24.52
1888	39.24	-6.85	GA	10.98	NY	50.35	25.80
1896	38.04	-7.97	GA	11.21	IL	47.47	26.55
1900	36.64	-7.33	SC	13.40	NY	51.56	25.37
1904	32.76	-9.82	TX	18.81	NY	44.68	26.08
1908	33.39	-9.10	GA	13.87	OH	43.84	24.70
1916	31.70	-9.82	GA	34.22	IL*	44.43	25.99
1920	32.72	-11.53	TX	17.22	OH	44.57	30.33
1928	33.86	-10.66	GA	17.46	NY	44.74	24.77
1932	34.97	-11.09	TX	16.62	IL	39.98	27.14
1936	32.54	-13.01	TX	18.42	NY	40.25	25.08
1940	32.73	-11.62	TX	20.23	NY	38.11	26.14
1944	31.89	-11.72	TX	23.15	NY	47.56	26.66
1952	36.40	-7.32	AL	16.18	NY	49.15	25.11
1956	35.83	-7.28	TX	16.11	NY	47.08	25.57
1964	39.24	-6.33	AL	13.96	CA	54.31	22.14
1972	42.06	-3.75	GA	11.93	CA	58.28	22.49
1976	43.06	-2.64	SC	6.29	CA	50.30	25.04
1984	44.10	-2.39	SC	7.89	CA	54.93	24.89
1988	45.82	-2.20	SC	10.88	CA	54.58	24.76
1992	44.03	-2.06	WY	4.82	IL	52.01	24.43
1996	45.37	-2.02	HW	4.77	FL	53.21	22.97
2000	46.90	-2.18	HW	5.89	FL	56.84	26.10
2004	43.30	-2.13	HW	6.11	FL	54.66	23.78
2008	43.53	-2.14	HW	7.61	FL	52.30	22.99
MEAN	38.42	-7.03		13.41		50.55	24.68

(1) Under perfect apportionment, the state would lose/gain this many electoral votes

*** In 1916, Illinois was the only large state with full women's suffrage**

Table 7. Inversion Intervals under House Apportionment

ELECTION	DEM 2-PARTY VOTE PERCENT	DEM HOUSE EV VOTE PERCENT	DEM HOUSE EV VOTE PERCENT AT PV = 50%	DEM HOUSE EV WIN PERCENT	DEM HOUSE EV INVERSION INTERVAL	ABSOLUTE HOUSE EV INVERSION INTERVAL
1828	56.19	74.18	44.60	50.65	-.65	.65
1836	50.87	61.57	59.09	49.68	.32	.32
1840	47.52	19.01	53.31	49.03	.97	.97
1844	50.74	62.78	47.53	50.20	-.20	.20
1848	47.33	42.17	66.09	49.22	.78	.78
1852	53.69	85.47	37.18	50.95	-.95	.95
1860	60.17	41.35	39.24	62.51	-12.51	12.51
1864	45.99	8.15	48.37	50.41	-.41	.41
1868	47.34	28.76	50.00	50.75	.00**	.00**
1872	44.07	18.49	59.93	47.64	2.36	2.36
1876	51.53	51.19	45.39	50.88	-.88	.88
1880	49.99	40.27	40.27	50.95	-.95	.95
1884	50.13	55.08	44.62	50.09	-.09	.09
1888*	50.41	40.62	35.08	50.97	-.97	.97
1896	47.81	36.41	45.66	50.23	-.23	.23
1900	46.83	33.89	41.18	51.14	-1.14	1.14
1904	39.99	28.24	51.81	49.83	.17	.17
1908	45.50	31.71	46.55	50.81	-.81	.81
1916*	51.64	49.66	45.98	51.46	-1.46	1.46
1920	36.17	24.14	46.44	52.74	-2.74	2.74
1928	41.20	16.32	57.47	48.63	1.37	1.37
1932	59.16	89.20	46.67	50.09	-.09	.09
1936	62.45	99.08	40.46	51.68	-1.68	1.68
1940	54.97	85.75	38.39	51.51	-1.51	1.51
1944	53.78	82.76	38.39	51.26	-1.26	1.26
1952	44.60	16.32	47.36	50.36	-.36	.36
1956	42.25	13.79	52.41	49.67	.33	.33
1964	61.34	91.03	57.01	48.40	1.60	1.60
1972	38.21	2.76	54.02	49.24	.76	.76
1976	51.05	57.24	48.74	50.19	-.19	.19
1984	40.80	1.84	45.52	50.30	-.30	.30
1988	46.10	20.69	48.28	50.08	-.08	.08
1992	53.46	69.89	50.34	49.72	.28	.28
1996	54.74	72.41	53.56	49.58	.42	.42
2000	50.23	51.72	46.67	50.15	-.15	.15
2004	48.76	48.74	54.59	49.16	-.84	.84
2008	53.69	70.34	52.87	48.82	1.18	1.18
MEAN***	49.18	46.71	48.36	50.18	-.21	.79

* Actual election inversion
** A 'spanning' tie interval from 49.54% to 50.75%
*** Excluding 1860

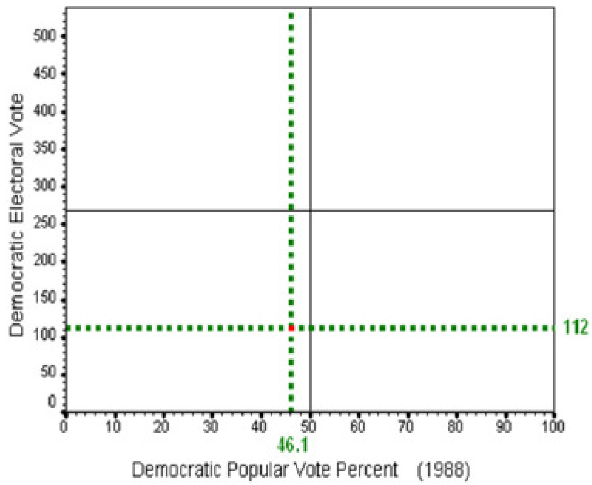
Table 8. Inversion Intervals under Perfect Apportionment

ELECTION	DEM 2-PARTY VOTE PERCENT	DEM PERF APPORT EV PERCENT	DEM PERF APPORT EV PERCENT AT PV = 50%	DEM PERF APPORT WIN PERCENT	DEM PERF APPORT INVERSION INTERVAL	ABSOLUTE PERF APPORT INVERSION INTERVAL
1828	56.19	76.95	35.65	54.61	-4.61	4.61
1836	50.87	55.85	53.36	49.68	.32	.32
1840	47.52	16.18	54.33	49.32	.68	.68
1844	50.74	61.84	44.19	50.20	-.20	.20
1848	47.33	42.16	70.41	49.22	.78	.78
1852	53.69	91.92	32.65	50.95	-.95	.95
1860	60.17	32.71	30.13	62.51	-12.51	12.51
1864	45.99	5.90	49.55	50.41	-.41	.41
1868	47.34	27.83	51.43	49.54	.46	.46
1872	44.07	15.97	61.16	47.64	2.36	2.36
1876*	51.53	49.37	42.87	51.78	-1.78	1.78
1880	49.99	34.67	34.67	50.95	-.95	.95
1884	50.13	51.16	39.56	50.09	-.09	.09
1888*	50.41	33.85	28.31	50.97	-.97	.97
1896	47.81	29.44	40.10	50.23	-.23	.23
1900	46.83	24.30	33.37	50.22	-.22	.22
1904	39.99	13.65	44.68	49.83	.17	.17
1908	45.50	21.90	39.52	52.09	-2.09	2.09
1916*	51.64	44.93	38.56	52.15	-2.15	2.15
1920	36.17	12.05	41.10	56.59	-6.59	6.59
1928	41.20	8.00	49.38	50.07	-.07	.07
1932	59.16	89.46	37.36	52.38	-2.38	2.38
1936	62.45	99.01	28.40	52.29	-2.29	2.29
1940	54.97	83.63	26.64	52.77	-2.77	2.77
1944	53.78	79.10	25.84	51.82	-1.82	1.82
1952	44.60	9.53	41.98	50.41	-.41	.41
1956	42.25	8.05	47.08	51.48	-1.48	1.48
1964	61.34	94.41	59.49	48.40	1.60	1.60
1972	38.21	3.42	58.28	48.38	1.62	1.62
1976	51.05	55.31	46.81	50.19	-.19	.19
1984	40.80	2.48	45.93	50.30	-.30	.30
1988	46.10	21.05	49.08	50.05	-.05	.05
1992	53.46	71.44	50.59	49.72	.28	.28
1996	54.74	72.98	53.21	49.58	.42	.42
2000	50.23	51.10	45.42	50.15	-.15	.15
2004	48.76	48.26	54.66	49.16	.84	.84
2008	53.69	71.32	52.30	48.84	1.16	1.16
MEAN**	49.18	43.85	44.66	50.62	-.62	1.37

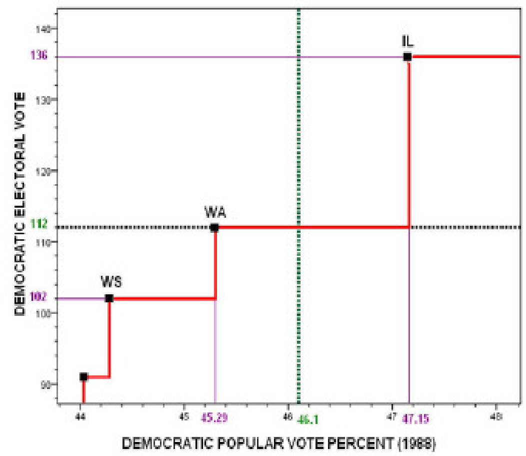
* Actual election inversion
** Excluding 1860

Table 9. Magnitude and Direction of Distrubution and Apportionment Effects

ELECTION	DEM ELECTORAL VOTE WIN PERCENT	NO DISTRIBUTION OR APPORTIONMENT EFFECTS	DEM INV INT DUE TO DISTRIBUTION EFFECTS	DEM INV INT DUE TO APPORTIONMENT EFFECTS
1828	50.65	50.00	5.26	-4.61
1836	49.68	50.00	-0.64	0.32
1840	49.32	50.00	-1.36	0.68
1844	50.20	50.00	0.40	-0.20
1848	49.22	50.00	-1.56	0.78
1852	50.95	50.00	1.90	-0.95
1860	61.26	50.00	23.77	-12.51
1864	50.41	50.00	0.82	-0.41
1868	50.75	50.00	0.29	0.46
1872	48.26	50.00	-4.10	2.36
1876	51.78	50.00	3.56	-1.78
1880	50.95	50.00	1.90	-0.95
1884	50.09	50.00	0.18	-0.09
1888	50.97	50.00	1.94	-0.97
1896	50.23	50.00	0.46	-0.23
1900	51.14	50.00	1.36	-0.22
1904	49.83	50.00	-0.34	0.17
1908	50.81	50.00	2.90	-2.09
1916	51.46	50.00	3.61	-2.15
1920	51.81	50.00	8.40	-6.59
1928	48.63	50.00	-1.30	-0.07
1932	50.09	50.00	2.47	-2.38
1936	51.68	50.00	3.97	-2.29
1940	51.51	50.00	4.28	-2.77
1944	51.26	50.00	3.08	-1.82
1952	50.36	50.00	0.77	-0.41
1956	49.67	50.00	1.15	-1.48
1964	48.97	50.00	-2.63	1.60
1972	49.70	50.00	-1.92	1.62
1976	50.19	50.00	0.38	-0.19
1984	50.83	50.00	1.13	-0.30
1988	50.08	50.00	0.13	-0.05
1992	50.65	50.00	0.37	0.28
1996	49.58	50.00	-0.84	0.42
2000	50.27	50.00	0.42	-0.15
2004	49.81	50.00	-1.03	0.84
2008	49.14	50.00	-2.02	1.16



(a)



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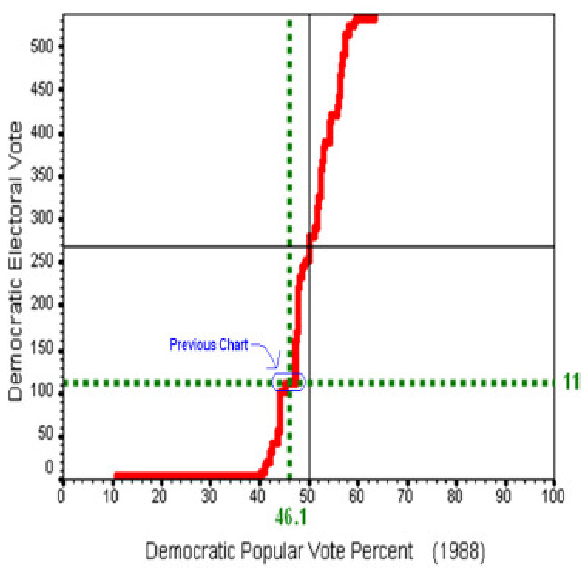

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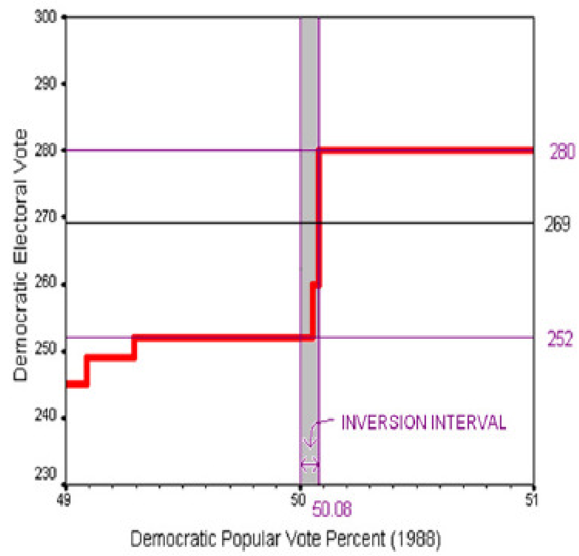
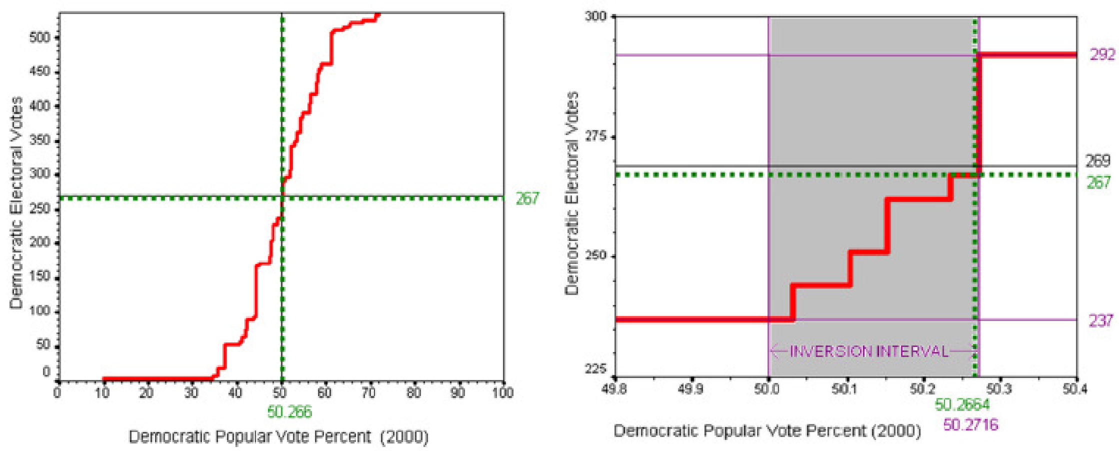


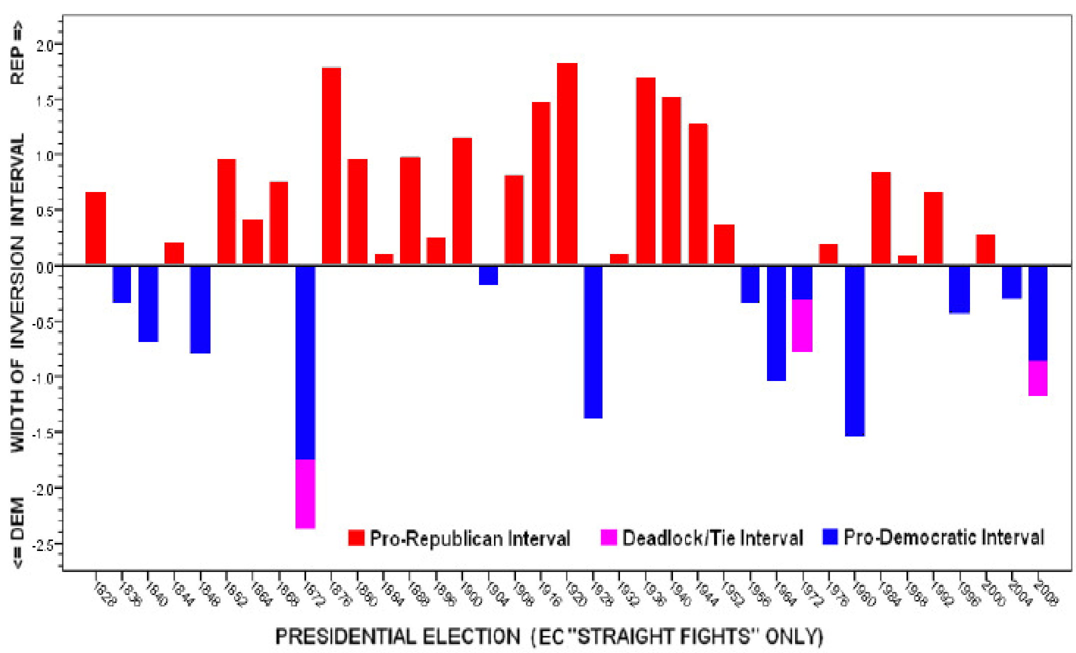
(d)

Figure 1



(a) (b)

Figure 2



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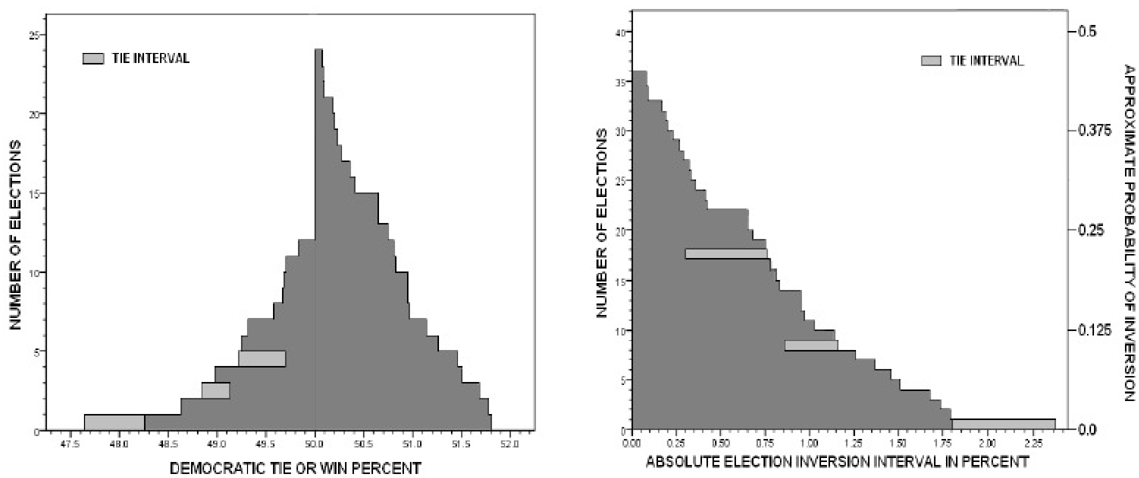
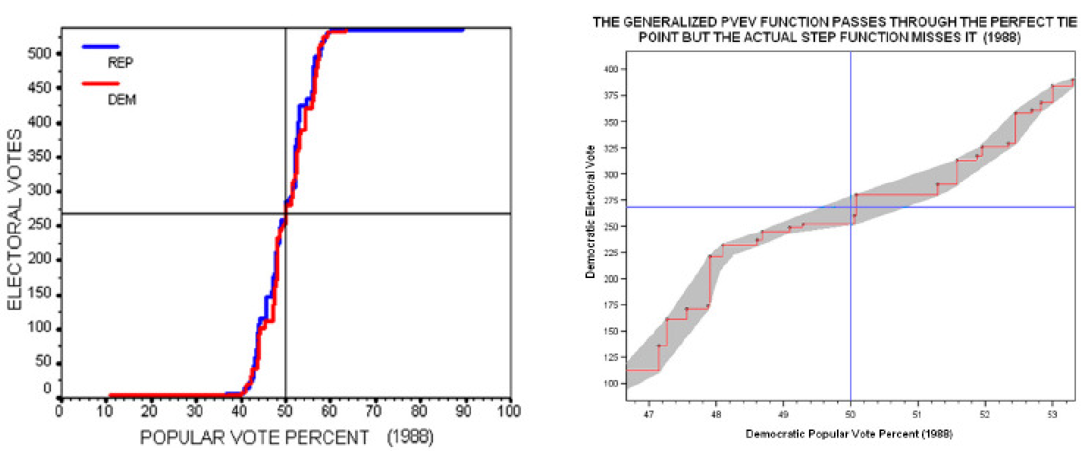


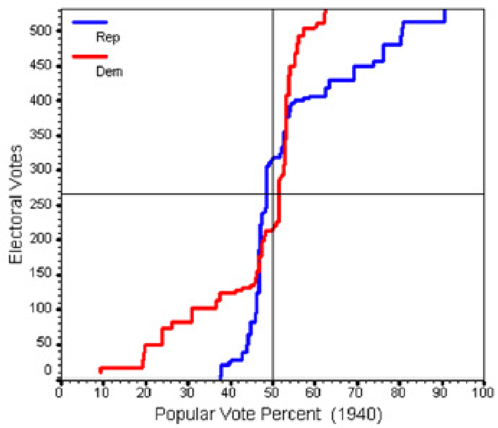
(a) (b)

Figure 4



(a) (b)

Figure 5



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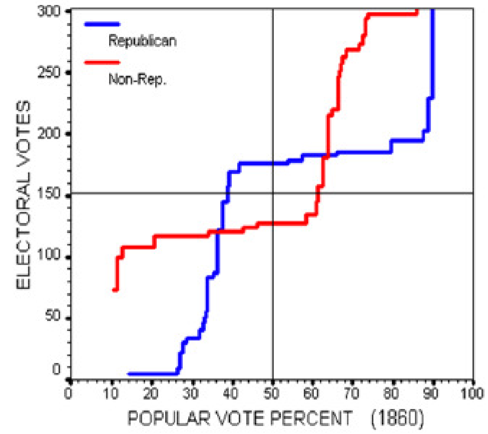
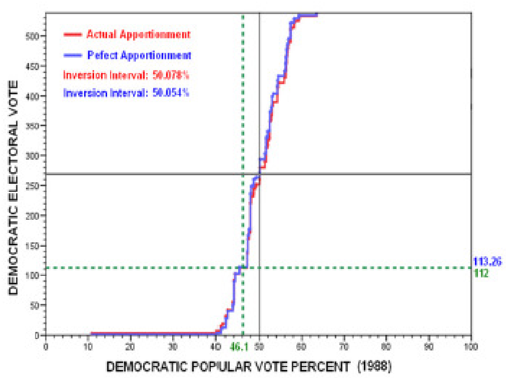


(b)

Figure 6



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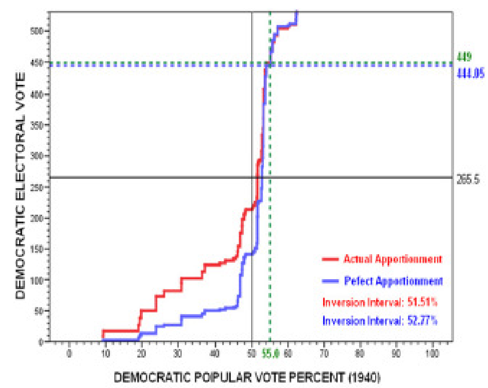

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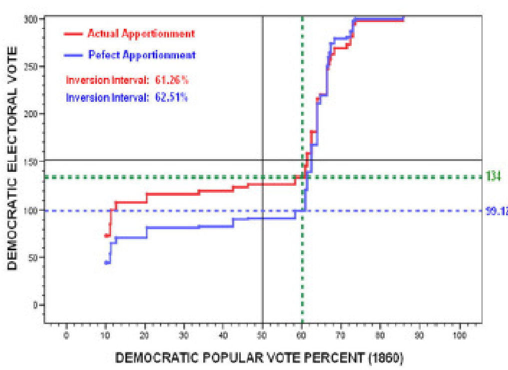


(c)

Figure 7

(a)

(b)

(c)

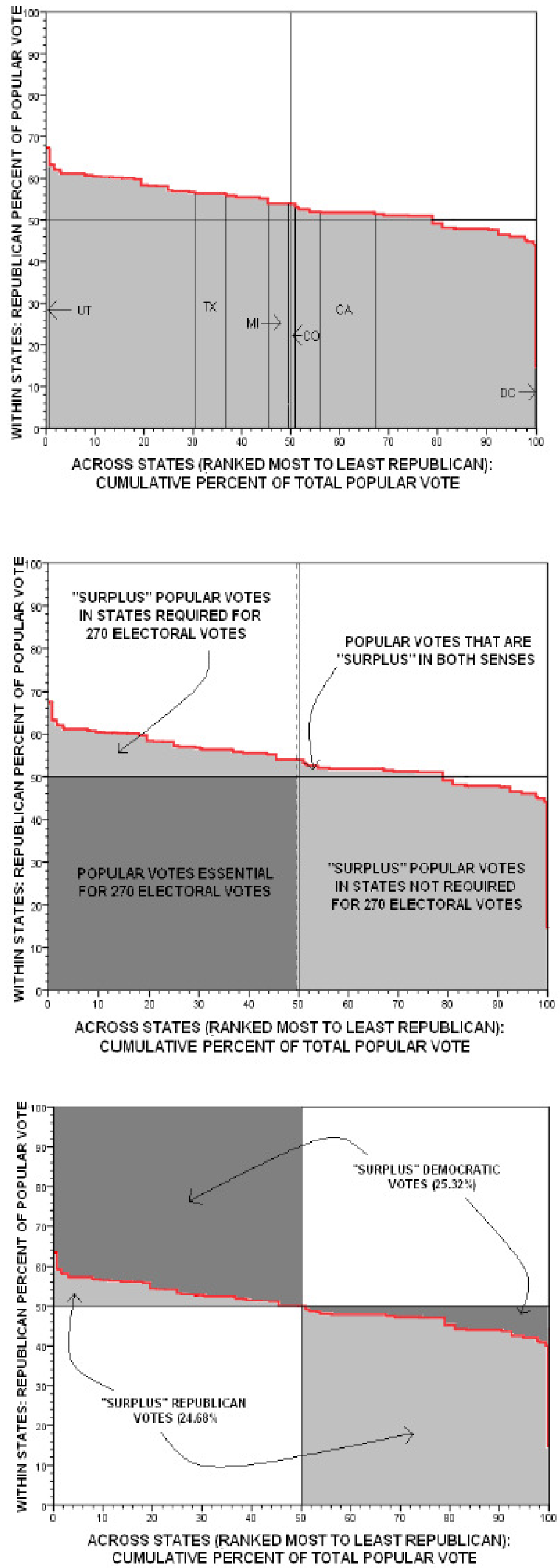
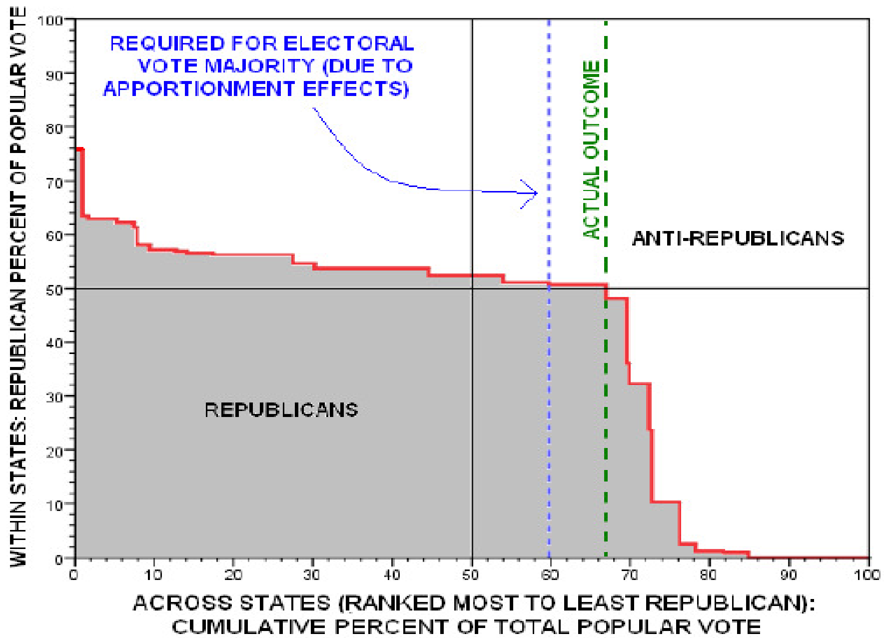
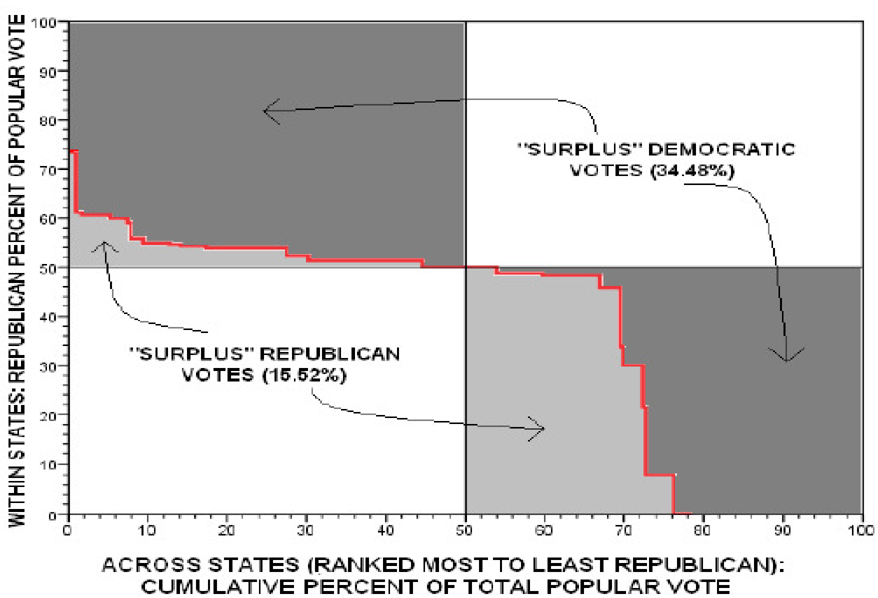

WITHIN STATES: REPUBLICAN PERCENT OF POPULAR VOTE
UT
TX
MI
CO
CA
DC
ACROSS STATES (RANKED MOST TO LEAST REPUBLICAN): CUMULATIVE PERCENT OF TOTAL POPULAR VOTE
"SURPLUS" POPULAR VOTES IN STATES REQUIRED FOR 270 ELECTORAL VOTES
POPULAR VOTES THAT ARE "SURPLUS" IN BOTH SENSES
POPULAR VOTES ESSENTIAL FOR 270 ELECTORAL VOTES
"SURPLUS" POPULAR VOTES IN STATES NOT REQUIRED FOR 270 ELECTORAL VOTES
"SURPLUS" DEMOCRATIC VOTES (25.32%)
"SURPLUS" REPUBLICAN VOTES (24.68%)

Figure 8


REQUIRED FOR ELECTORAL VOTE MAJORITY (DUE TO APPORTIONMENT EFFECTS)
ACTUAL OUTCOME
ANTI-REPUBLICANS
REPUBLICANS
WITHIN STATES: REPUBLICAN PERCENT OF POPULAR VOTE
ACROSS STATES (RANKED MOST TO LEAST REPUBLICAN): CUMULATIVE PERCENT OF TOTAL POPULAR VOTE

(a)


"SURPLUS" DEMOCRATIC VOTES (34.48%)
"SURPLUS" REPUBLICAN VOTES (15.52%)
WITHIN STATES: REPUBLICAN PERCENT OF POPULAR VOTE
ACROSS STATES (RANKED MOST TO LEAST REPUBLICAN): CUMULATIVE PERCENT OF TOTAL POPULAR VOTE

(b)

Figure 9

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