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Nicholas R. Miller
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Power to Initiate Action and Power to Prevent Action

These terms, which pertain to the general topic of **power indices**, were introduced by James S. Coleman in a paper on the "Control of Collectivities and the Power of a Collectivity to Act" (1971).

Coleman observed that the **Shapley-Shubik power index** (1954) — the most commonly used measure of voting power at the time — is based on cooperative game theory and assumes that players seek to form a winning coalition whose members divide up some fixed pot of spoils. "But the situation posed by decisions in collective bodies is ordinarily quite different. The decision governs an action to be taken — or not taken, depending on the outcome — by the collectivity, an action that has a fixed profile of consequences for the members" (1971, pp. 276-277). For example, if the members are choosing whether to provide themselves with a public good, there are no winnings to divide — regardless of how each member votes, he or she will bear the same consequences (though different members likely evaluate them differently).

Since a voter's expected share of the spoils itself measures voting power, Shapley-Shubik power within any group necessarily sums to one. But, Coleman observed, the power of collectivities to act may vary greatly. A body that uses unanimity rule to initiate action often fails to act because unanimity is difficult to achieve, while one that uses simple majority rule is in some sense as likely to act as not. While voting power *within* each body is equal, in a meaningful sense *all* members of the majority-rule group are more powerful than those in the unanimity-rule group. This kind of example led Coleman to distinguish between two apparently distinct but closely interrelated "faces"of individual voting power: the "power to initiate action" and the "power to prevent action."

In describing how Coleman formalized these concepts, it is useful to have at hand a specific example of a five-member weighted voting body. By voting 'yes' or 'no,' 5 voters can partition themselves into 16 complementary subsets (or "coalitions"), so altogether there are 32 distinct coalitions (including the coalition of all and its empty complement), and each voter belongs to half of them. Suppose that voter A has 3 votes, B has 2 votes, and C, D, and E have 1 vote each and that a *quota* of 5 votes is required to initiate action (from which it follows that 4 votes is sufficient to prevent action). It can be checked that there are five *minimal winning coalitions,* each of which has the required 5 votes to initiate action but would fall below the quota if *any* member left the coalition: {A,B}, {A,C,D}, {A,C,E}, {A,D,E}, and {B,C,D,E,}. It can be further checked that eight additional coalitions have more than 5 votes, giving a total of 13 winning coalitions. A voter i is *critical* to a coalition S if i belongs to S and S is winning but $S - \{i\}$ is not (or, equivalently, if i does not belong to S and S is not winning but $S - \{i\}$ is). Every member of a minimum winning coalition is critical to it, but some or all members of more inclusive coalitions fail to be critical. It can be checked that A is critical for 11 winning coalitions, B for 5, and C, D, and E for 3 each.

Coleman's **power of the collectivity to act** A_N is simply the fraction of all coalitions that are winning; in our example, A_N = 13/32 = .4063. With respect to an individual voter i, Coleman's ***power to prevent action*** P_i is the fraction of winning coalitions for which i is critical; in our example, $P_A = 11/13 = .8462$, $P_B = 5/13 = .3841$, and $P_C = P_D = P_E = 3/13 = .2308$. Coleman's ***power to initiate action*** is I_i is the fraction of non-winning coalitions for which *i* is critical; in our example, $I_A = 11/19 = .5790$, $I_B = 5/19 = .2632$, and $I_C = I_D = I_E = 3/19 = .1579$.

While they may seem to lack theoretical justification, these fractions have intuitive and meaningful interpretations in terms of probability. If we know nothing about the voting situation other than its formal rules, our *a priori* expectation must be that everyone votes randomly, i.e., as if independently flipping fair coins. In this *random voting* or *Bernoulli model*, each partition of voters into complementary sets has equal probability of occurring and each coalition in each complementary pair is equally likely to vote 'yes,' so: (i) the power of the collectivity to act is the probability that the collectivity acts; (ii) voter *i*'s power to prevent action is the probability that, given the action is initiated with *i*'s support, *i* can prevent it by switching his vote to 'no'; and (iii) *i*'s power to initiate action is the probability that, given that the action is prevented with *i* in the opposition, *i* can initiate it by switching his vote to 'yes.'

It is surprising that Coleman did not propose an overall measure of *individual voting power* D_i, since one readily suggests itself — namely, the fraction of all coalitions to which *i* belongs for which *i* is critical; in our example, $D_A = 11/16 = .6875$, $D_B = 5/16 = .3125$, and $D_C = D_D = D_E = 3/16 = .1875$. D_i is the probability that, whatever the outcome of a vote, voter *i* can reverse it by switching his vote — that is, the probability that *i* casts a *decisive* vote. If the collectivity has maximum power to act (i.e., if half of all coalitions are winning), as under majority rule with an odd number of voters, $P_i = I_i = Di$; otherwise D_i is the harmonic mean of P_i and I_i and, provided complementary coalitions cannot both be winning, $I_i < D_i < P_i$.

Evidently, Coleman was unaware of the **Banzhaf (or Penrose-Banzhaf) power index** that had been proposed a few years earlier (Banzhaf, 1965; also see Penrose, 1946) but D_i is simply the **absolute Banzhaf power measure** and, if P_i, I_i, and D_i are rescaled so that the power of all voters adds up to 1, they are equivalent to one another (because they have the same numerators) and to the **relative Banzhaf index**. Just as the absolute Banzhaf measure is more informative than the relative Banzhaf index, the two individual Coleman measures are more informative than the absolute Banzhaf measure alone.

Coleman's analysis of voting power attracted relatively little attention, but in their recent comprehensive and (perhaps) definitive history and analysis of *a priori* voting power concepts, Felsenthal and Machover (1998, 2005) give considerable credit to Coleman's analysis — especially his conceptual critique of the Shapley-Shubik index — for clarifying the differing theoretical underpinnings of the Shapley-Shubik and Banzhaf measures.

References

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