Agenda trees and sincere voting: a response to Schwartz

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Abstract Schwartz (Public Choice 136:353–377, 2008) has identified a controversy within the voting theory literature pertaining to the representation of agenda structures and the consequent definition of sincere voting. This note responds to Schwartz's remarks by arguing that the kind of agenda tree he uses does not adequately represent some common parliamentary agendas, and that consequently his definition of sincere voting cannot always be applied.

In his authoritative analysis of parliamentary voting procedures and their effects on political outcomes, Schwartz (2008) identifies a controversy within the voting theory literature pertaining to the representation of agenda structures and the consequent definition of sincere voting. Since Schwartz identifies me as a party to the controversy, a response is in order. I first review the relevant background and then address the controversy by focusing on Schwartz's Appendix 2 and his accompanying Fig. 8, which is reproduced here as Fig. 1.

The relevant voting theory literature begins with Black (1948, 1958). My quick check indicates that, contrary to Schwartz's remark (2008, p. 356), Black never used the term "amendment procedure" (though he did note that one "motion" might be an "amendment" to another motion). Black (1948, p. 25) spoke of voting taking place in this manner:

> In practice, voting would be so conducted that, after discussion, one motion would be made and, after further discussion, another motion (an 'amendment,' that is) might be moved. If so, the original motion and amendment would be placed against each other in a vote. One of the two motions having been disposed of, leaving a single motion in the field, a further amendment to it might be moved; then a further vote would be taken between the survivor of the first vote and the new motion; and so on.

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Black ([1958](#)) referred to this manner of voting initially ([1948](#), p. 21) as Procedure (α) and subsequently ([1948](#), p. 44–45) as "ordinary committee procedure." His language indicates that alternatives are explicitly paired at each vote and that, at the time of each pairwise vote, voters do not know what other alternatives may subsequently be proposed for a vote. Except for making no explicit reference to the status quo (as the first "motion"), Black's "ordinary committee procedure" is equivalent to what Wilson ([1986](#), p. 393, emphasis added) later called a "forward-moving agenda," in which alternatives are voted on as they are proposed.

> This process begins with an amendment offered to the status quo. If the amendment wins, it becomes the new status quo, and further amendments can be proposed. This process continues until discussion is halted, with the current status quo declared the collective choice. *This process is typically modeled by social choice theorists but it has few counterparts in a natural setting.* Such processes are limited to small decision bodies which do not have formal parliamentary rules.[1]

Wilson contrasted such a forward-moving agenda with a backward-built one, in which all alternatives are proposed before any voting takes places, and alternatives are then voted on in the reverse of the order in which they proposed.

> A backward voting procedure requires that the status quo be voted on last and that the agenda be ordered. Under this procedure, a bill is offered to the floor. But, before voting, amendments can be offered to the bill, as well as to any amendments. Voting is in reverse order, with the final amendment which is offered voted on first, and the bill (as amended) voted last. At the final vote, the choice is between the (amended) bill and retaining the status quo. *This procedure resembles greatly that used in the United States Congress as well as in other legislative settings.*

> In his book ([1969](#)) and in earlier articles ([1956a](#) and [1956b](#)), Farquharson characterized a voting procedure (i.e., an agenda) in this way ([1969](#), p. 9):

> We shall define a *voting procedure*, initially, in terms of the set of outcomes. Suppose this set is divided into two subsets, each subset into two further subsets, and so on until single outcomes are reached. Such a sequence of divisions may be depicted as a tree, the "outcome tree." Each of its forks corresponds to a division, and the end of each of its branches corresponds to an outcome.

For each division of a set into two subsets, Farquharson ([1969](#), p. 11) required the following:

AXIOM I. Each element of the set be in at least one subset.
AXIOM II. Neither subset coincide with the whole set.

Such a sequence of divisions can be represented by a binary tree in the manner of Fig. [1](#)b. While Farquharson drew his figures in the manner of Fig. [1](#)c (with colored balls, not letters, representing outcomes), Fig. [1](#)b contains no information that is not implied in Fig. [1](#)c (as Schwartz notes). Farquharson assumed that a voting agenda is fully built (i.e., all alternatives have been proposed) before any voting take place and that the agenda is known to all voters.

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[1]The story associated with McKelvey's ([1976](#)) Global Cycling Theorem—that an agenda setter can (almost always) design a sequence of pairwise votes leading from any status quo to any other point in a space of two or more dimensions—uses a forward-moving agenda.
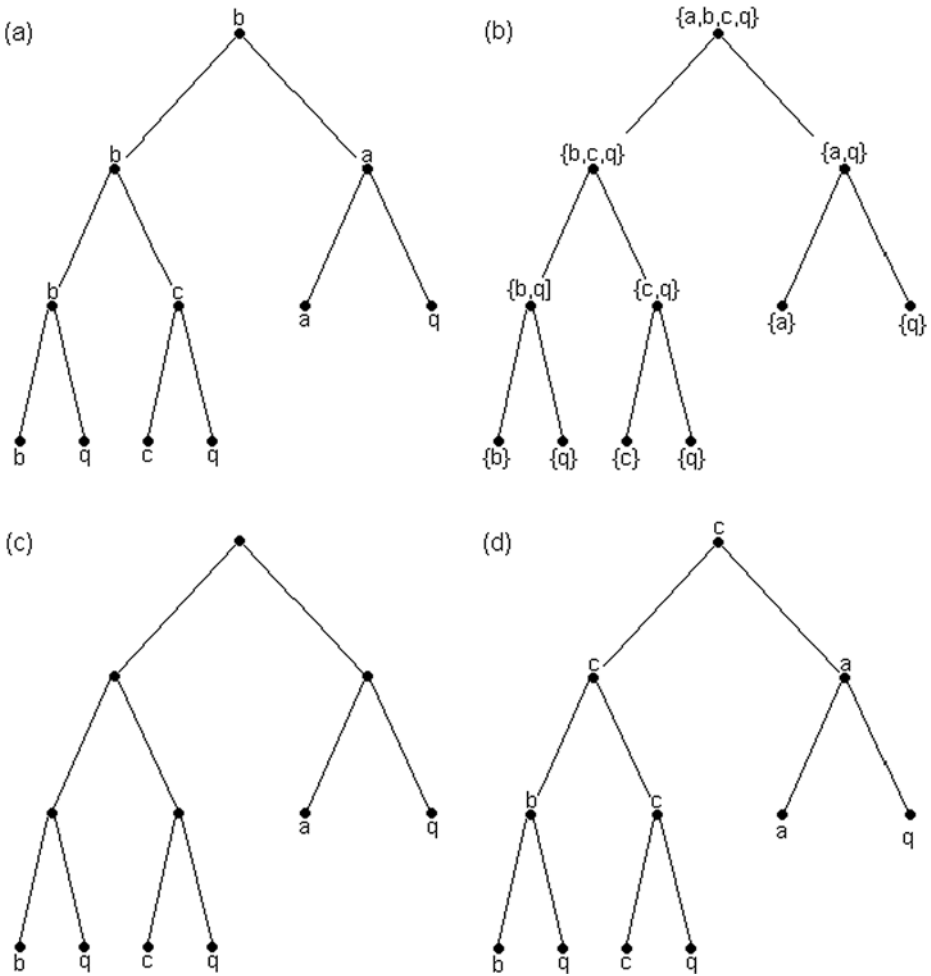
Fig. 1 Schwartz's Fig. 8

Farquharson identified two specific binary voting procedures (1969, p. 61), which he dubbed *successive* (essentially equivalent to Schwartz's "Euro-Latin" procedure and a procedure never considered by Black) and *amendment* (equivalent to the restrictive "amendment agenda" variant of Schwartz's "Anglo-American" procedure and similar to Black's "ordinary committee procedure" except that all motions are introduced and ordered for voting before any voting takes place). Farquharson, like Black, did not consider the parliamentary status of different alternatives and, in particular, did not assign a special role of the status quo alternative.

Black generally assumed that committee members would vote "in accordance with their schedules of preference" (i.e., in the manner that Farquharson would later dub "sincere voting"), but this characterization is directly applicable only if (as Black assumed) alternatives are explicitly paired for votes. In contrast, Farquharson assumed that an agenda is built before voting begins, and he defined "sincere" voting in terms of what Schwartz (2008, p. 355)

calls "lexicographic maximax" voting and, more famously, he defined "sophisticated" (or strategic) voting in terms of "ultimately admissible" voting strategies.[2]

Thereafter, many scholars (Banks 1985, 1989; Bjurulf and Niemi 1982; Feld et al. 1989; Jung 1990; Miller 1977, 1980; Niemi and Gretlein 1985; Niemi and Rasch 1987; Shepsle and Weingast 1984) used Farquharson's definitions and tools to derive propositions concerning sincere and strategic voting outcomes under these two procedures (with the greater attention on amendment procedure), and still (for the most part) taking no special account of the status quo or the parliamentary status of alternatives.

Ordeshook and Schwartz (1987) criticized this literature on the grounds that Farquharson's stylized version of "amendment procedure" did not encompass many actual agendas found in Anglo-American legislatures, let alone elsewhere, and they emphasized that the special role of the status quo had to be recognized. Ordeshook and Schwartz's analysis greatly broadened the scope of agenda structures, and they identified different formal properties that parliamentary agendas might display and examined their impact on both sincere and strategic voting outcomes. Along the way, they modified Farquharson's definition of an agenda tree by associating a unique alternative (rather than a subset of the alternatives) with each node (division) in the agenda tree (in the manner of Figs. 1a and 1d, and in contrast to Figs. 1b and 1c). This in turn allowed them to employ a simpler and more direct definition of sincere voting—namely, that a sincere voter chooses his preferred alternative at each node.

In my own later work (1995) on agenda structures, I considered a wide range of agenda structures and adopted some of Ordeshook and Schwartz's concepts and much of their terminology. I first (1995, pp. 10–18) worked through a number of agenda examples that took explicit note of the status quo and the parliamentary status of alternatives. I noted that many agendas *implicitly* pair alternatives in a sequence of votes. But I also noted (1995, p. 16) that, under "incomplete" agendas, "the informal specification of the agenda in terms of pairwise votes is not entirely appropriate" and I promised that I would subsequently introduce "an analytic device that both identifies and sidesteps this ambiguity." That device was the agenda tree, defined in the manner of Farquharson. In a footnote (1995, p. 18), I explicitly noted that Ordeshook and Schwartz's "method of representing an agenda structure differs from [what I judged to be] the more standard one presented here." When a set of alternatives is divided into two subsets, I called the alternatives in the intersection of the two subsets *unchallenged* at that division, in that they will remain as possible outcomes after the vote is taken regardless of the result of the vote. Under a *pairwise* agenda, there are exactly two challenged alternatives, one in each subset, so the two challenged alternatives are implicitly paired for a vote at each division. Farquharson's "amendment procedure" is pairwise, as are many but not all variants of it, but his "successive procedure" is not. In a further footnote (1995, p. 27), I observed that "the substantive assumption behind [Ordeshook and Schwartz's] definition is that all binary agendas are effectively pairwise, but this supposition seems hard to justify."

In (1977, 1980, and 1995), I also adopted Farquharson's definition of sincere voting, which I characterized informally in this way (1995, p. 45, emphasis added):

> *If alternatives are explicitly paired for votes*, the notion of sincere voting is intuitively clear, at least for a voter with strong preferences: *a sincere voter always votes for his more preferred alternative*. But not all agendas are pairwise (or even binary). For nonrepetitive sequential binary procedures, Farquharson (1969) developed the following

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[2]McKelvey and Niemi (1978) subsequently redefined strategic voting in terms of multistage games in a way that made its analysis far more tractable.

notion of sincerity: a sincere voter has (often excessively) "optimistic" expectations, always aiming for the best possible outcome.

Put otherwise, at each division a sincere voter votes for the subsequent node associated with his most preferred challenged alternative. In a footnote, I further observed (1995, p. 46) that

> Ordeshook and Schwartz (1987) define sincere voting differently, in terms of their definition of an agenda tree. ... Since a pairwise agenda tree is compatible with only one Ordeshook–Schwartz agenda, the two definitions are equivalent in this case. But if an agenda fails to be pairwise due to incompleteness, the two definitions can label different choices—and thus different strategies—as sincere.

So, the controversy concerning agendas trees and sincere voting was clearly identified some years ago. I now address the merits of the controversy, focusing on the example presented in Fig. 1. All the agenda trees in Fig. 1 represent a situation in which a bill b has been proposed along with an amendment a and a "backup" amendment c to be considered only if a is rejected.[3]

First, observe that the agenda depicted in Fig. 1 is *not* pairwise: three alternatives—all but q—are challenged at the first vote. Schwartz (2008, pp. 375–376) notes that

> the Miller–Farquharson tree [Fig. 1b or Fig. 1c] omits information. It does not tell us which two alternatives are compared at any but a final vote.... The cost shows up in [Fig. 1d]. It obviously differs from [Fig. 1a] in its information content—in which alternatives are compared at the first vote. Yet Farquharson and Miller would represent both [Fig. 1d and 1a] by [Fig. 1b], or equivalently [Fig. 1c].

I do not dispute that Farquharson and Miller would represent both Figs. 1a and 1d by Figs. 1b or 1c and that the latter contain less information than the former. But I do question *whether the information that is given in Figs.* 1a *and* 1d, *but is missing from Figs.* 1b *and* 1c, *is information that voters really have and can act on* (sincerely or otherwise).

To the best of my knowledge, parliamentary voting procedures for voting on motions (in contrast to electoral procedures for voting on candidates) almost never *explicitly* pair alternatives for votes. Rather they prescribe a series of "questions" pertaining to the adoption of motions on which *yea* and *nay* votes are taken. Under Anglo-American procedure, the proposals in play in Fig. 1 raise two or three questions to be voted on, the number depending on the outcome of the vote on the first question.

Question 1. Shall the amendment a be adopted? *If the yeas have it*:
Question 2. Shall the bill as amended by a be adopted?

Question 1. Shall the amendment a shall be adopted? *If the nays have it*:
Question 2. Shall the amendment c be adopted?
Question 3. Shall the bill (as amended by c or not) be adopted?

When I stare at Fig. 1, I cannot see that Fig. 1a is clearly more appropriate than Fig. 1d (or vice versa) for representing the sequence of votes on these questions. Conversely, I can't see how to distinguish between the *parliamentary situations* (as opposed to explicit pairings of alternatives) represented by Figs. 1a and 1d. What is clear is that, if the yeas have it on the Question 1, both c and b are eliminated as possible outcomes and, if the nays have it, a is eliminated—exactly what is represented in Figs. 1b and 1c.

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[3]In Fig. 1 (as in Schwartz's Fig. 8), a and c actually represent the prospective bill modified by one or other amendment.

A sincere voter who prefers *c* to *a* to *b* certainly might "frame" Question 1 in the manner of Fig. 1a, i.e., as a choice between *b* and *a* ("do I prefer the bill with or without the original amendment?") and, therefore, vote yea on the question, reflecting his preference for *a* over *b*. But the voter might instead "frame" Question 1 in the manner of Fig. 1d, i.e., as a choice between *c* and *a* ("supposing the bill is to be amended, do I prefer the original or backup amendment?") and, therefore, vote nay on the question, reflecting his preference for *c* over *a*. A sincere voter with such preferences therefore confronts something of a dilemma concerning how to vote on Question 1—a dilemma which is reflected in Figs. 1b and 1c but not Figs. 1a and 1d.

More generally, while a node in an agenda tree resulting from a yea vote can be associated with a single alternative, a node resulting a nay vote often does not admit such a clear interpretation. This point was noted by Groseclose and Krehbiel (1993, p. 267, emphasis in original) some years ago.

> The meaning of nay votes is fundamentally ambiguous. In this case, is a nay at the first vote really a yea vote for the [backup] amendment [*c* or for the unamended bill *b*]? If we listen only to the parliamentarian, we absolutely cannot tell: he never says (as do voting theorists) "The vote is on the amendment [*a*] ... *versus the bill* [*b*]" [or "on the amendment *a versus the backup amendment c*"]. In other words, when as theorists we label the nay nodes of a binary tree, we in effect supply information about the voting situation that parliamentarians over the ages have not seen fit to supply.[4]

A response to this point might be that, as a backup amendment (and somewhat in the manner of a forward-moving agenda), *c* cannot be introduced and formally placed on the agenda unless and until Question 1 is voted down, so the second way of "framing" Question 1 is inappropriate because the voter does not know that backup amendment *c* is in the offing. But in Farquharson's setup voters do know the full agenda in advance and, if we do not follow Farquharson in this respect and instead assume that the prospective agenda is not generally known, neither Farquharson's nor Schwartz's analysis of strategic voting is applicable, and Schwartz's observation (2008, p. 375) that the differences among the agenda trees in Fig. 1 have no implications for strategic voting does not hold.

I therefore conclude that agendas of a parliamentary type, in which (i) all alternatives are (at least implicitly) proposed and placed on the agenda before voting begins and then (ii) voting proceeds through a sequence of yea or nay votes, are best represented by agenda trees of the Farquharson type, because the assignment of alternatives to nay nodes, as required by agenda trees of the Ordeshook–Schwartz type, may be inherently arbitrary. However, given a pairwise agenda, the two representations of agendas are effectively equivalent, with the latter displaying only the single unchallenged alternative in each subset of alternatives displayed in the former.

This leaves us with the question of how to characterize "sincere" voting. As previously noted, Black (1958) speaks of "committee members voting in accordance with their schedules of preference," a characterization is that unambiguous only if alternatives are explicitly

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[4]I have added the bracketed insertions to make Groseclose and Krehbiel's quote apply to the example at hand. The example to which they actually refer involved a bill, an amendment, and a substitute amendment, which together generate a (pairwise) amendment agenda in the restrictive Farquharson sense. In Groseclose and Krehbiel's view (though not Farquharson's or mine), assigning single alternatives to (nonterminal) nodes resulting from nay votes is more or less arbitrary even in this case. Groseclose and Krehbiel seek a working definition of sincerity in order to examine the pervasiveness of "sophisticated sincerity" in the sense of Austen-Smith (1987).

paired at each vote. Farquharson (1969) begins his discussion of sincere voting by para-
phrasing Black: "The simplest assumption that can be made about the behavior of voters is
that their votes are directly in accordance with their preference scales." But since he assumes
the agenda is built before voting begins and considers non-pairwise (e.g., successive) and
even nonbinary agendas, Farquharson must refine his characterization, and he does so in the
"minimax" fashion previously noted here and by Schwartz.

Schwartz (2008, p. 376) says: "The evident principle is this: A sincere voter votes for his
most preferred of the alternatives available for voting." But this formulation leaves the scope
of "availability" open. For Schwartz (as for Black with forward-moving "ordinary commit-
tee procedure"), availability is "local"—even with an extensive agenda known in advance,
only the two alternatives (allegedly) associated with each of the two nodes following a given
division are available at that particular vote. Farquharson (and Miller) can invoke the same
evident principle, but their availability is "global," encompassing all alternatives that remain
as possible voting outcomes.

"Sincere voting" is sometimes characterized as "shortsighted" (or "myopic") and "so-
phisticated" (or "strategic") voting as "farsighted," but there are two quite independent ways
in which a voter may be shortsighted or farsighted—namely, with respect to (i) what he
knows about the voting agenda and (ii) what he knows about the preferences of other voters.
If a voter is farsighted in both respects, he can "look ahead and reason back" and vote in
the "sophisticated" manner of Farquharson (or, more particularly, of McKelvey and Niemi
1978). Schwartz's sincere voter is shortsighted in the first respect ("he does not look beyond
his two voting options to the bottom of the tree"—though, I have argued, he may have dif-
ficulty identifying these two voting options) as well as the second. But, as Schwartz says,
Farquharson's (and Miller's) sincere voter is farsighted in the first respect ("like a strate-
gic legislator, he compares [the two alternatives available for voting at a given division] by
looking ahead to all their reachable consequences at the bottom of the tree") but (unlike a
strategic legislator) is shortsighted (indeed, blind) in the second respect.[5]

As Schwartz further observes, a sincere voter of the Farquharson type, having looked
ahead to the bottom of the tree, "compares [the two alternatives] on the basis of extreme
optimism rather than informed calculation [like a sophisticated voter]." But such extreme
optimism is not a necessary consequence of Farquharson's representation of an agenda tree.
In Fig. 1b, we can identify two different types of voting, both of which might be deemed
"sincere" in that they do not assume any knowledge of other voters' preferences. At the first
node in Fig. 1b, each voter is choosing between the subsets $\{c, b, q\}$ and $\{a, q\}$ or, if we
leave out the unchallenged alternative q, between $\{c, b\}$ and $\{a\}$. A voter who prefers c to a
to b would, by Farquharson's "maximax" definition of sincere voting, first identify the *best*
alternative in each subset (c and a, respectively) and then (by voting nay) chose the node
that associated with the better of the best. But a more prudent "minimax" type of sincere
voter would instead identify the worst alternative in each subset (b and a, respectively) and
then (by voting yea) vote for the node that contains the better of the worst.[6]

The general point, it seems to me, is that we should recognize that, under some (but not
all) voting procedures, there are different types of sincere voting and there is little point in

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[5]One possible characterization of a "sincere" voter is that he votes as if he were a *dictator* in the social choice
sense—or, more generally, as if he believes his vote will (somehow) be is decisive at every division—and
who therefore does not need to know anything about the preferences of other voters. Such a dictator would
be sincere in Farquharson's sense, not Schwartz's.

[6]Farquharson (1969, p. 18n) himself takes note the second possibility, which he characterizes as "prudential
voting."

arguing about which type is "truly" sincere. Moreover, any type of sincerity can result in regrettable choices. On the one hand, there is the example with which Schwartz concludes.

> To bring out the difference, suppose we at first have an Anglo-American agenda that pits *b* against *a*, then the winner again *q*. A whip needs your vote for *b* against *a* (not against *q*), but you love *a* and loathe *b*. Knowing that you are 'sincere' in the Farquharson–Miller sense, this whip shows you a new draft law, *c*, written to gratify your every wish, urge, ideal, dream, and interest. He promises to bring it to a vote if *b* defeats *a* at the first vote—a costlessly reliable promise, inasmuch as everyone but you abominates *c*. Thus expanded, the agenda is the first of Fig. 8 [Fig. 1a]. If Farquharson and Miller are right, you would vote for odious *b* against attractive *a*. How cheap they make your vote!

Well, yes—the whip's cajolery succeeds, because it allows me to be farsighted with respect to the agenda, while (evidently) keeping me blind to the preferences of other voters. It is true that I would do better voting for *a* on the first vote than voting against it, but I would realize this in advance only if I know enough about the preferences of other voters to realize that the promised vote on *c* is a hopeless prospect. But (as is true of sincere voters under either definition) I lack that knowledge.

On the other hand, suppose that the agenda is that depicted in Fig. 1a, that everyone is "sincere" in the Ordeshook–Schwartz sense, and that *everyone* loves *c* and despises the other two alternatives but regards *a* as marginally less despicable than *b*. If Ordeshook and Schwartz are right, everyone would vote for the moderately despicable *a* and thereby would forfeit the opportunity to enact the universally blissful *c*. How cheap they make everyone's votes.

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