POLI 388 Spring 2010

IN-CLASS MIDTERM TEST: ANSWERS AND DISCUSSION

1. A *game against nature* is a game between (i) a rational player whose interests/payoffs are at stake and (ii) a disinterested nature that makes choices in a random or quasi-random manner and that has no interests/payoff at stake. A *game of strategy* is a game between two (or more) rational players who both have payoff/interests (either common or conflicting) at stake.

2. Strategy S1 *dominates* strategy S2 (equivalently, S2 *is dominated by* S1) if S1 gives the player at least as good a payoff in every contingency (i.e., choice by nature or the other player) as S2 does and a better payoff in at least one contingency. S is a *dominant strategy* if it dominates every other strategy. The *dominance principle* (which applies to all types of games, including games against nature) says: never choose a dominated strategy (D&N, Rule 3) As a corollary, it says: use your dominant strategy if you have one (D&N, Rule 2). This is (for the most part) a very good decision principle so far as it goes. But it doesn't go very far, because typically players don't have dominant strategies. *Note*: if S1 dominates S2, what follows is that you should *not* choose S2, not that you should choose S1. (S1 may in turn be dominated by a third strategy or, in any event, you may have other undominated strategies.)

3. The *maximin principle* says that strategies should be evaluated in terms of their minimum payoffs (or *security levels*), i.e., assuming the worst, and that a player should choose the strategy with the highest security level, i.e., that gives the maximum of the minimums. Maximin strategies can be identified in all types of games (including games against nature). While the maximin principle provides (for the most part) good advice in a two-player *zero-sum game*; it is unduly pessimistic otherwise.

4. A *coordination game* is a two-player game in which the players have identical interests or payoffs (i.e., a zero-conflict game). Such a game is problematic if several outcomes (cells in the payoff matrix) serve the players common interests equally well. While, they have a common interest in coordinating their strategy choices, the players may have difficulty in doing so if they cannot communicate.

5. In a two-player game, a [Nash] *equilibrium* is a pair of strategies, one for P1 and one for P2, such that neither player has reason to regret his strategy choice, given the choice of the other player. Put otherwise,each strategy is a *best reply* to the other strategy. In a two-player zero-sum game, a Nash equilibrium is a cell with payoff (to the Pow player) that is both the maximum in its column (so the Row player can do no better) and the minimum in its row (so the Column player can do no better), which is sometimes called a *saddlepoint*.

6. The Battle of the Sexes refers to an "impure" (variable-sum) coordination game in which the players have a common interest in coordinating their strategy choices but conflicting interests as to *how* to coordinate them.

1 2	0 0
0 0	2 1

7. A two-player *zero-sum game* is a game in which the interests of the players are directly and invariably opposed. It is then possible to scale the payoffs to the two players so that the payoffs in each cell add up to zero. Thus only the payoffs to one player (by convention, the Row player) need to be shown (so the Column player seeks to minimize the displayed payoff). (The definition can be extended to games with more than two players; all common parlor [board and card] games are zero-sum.) A *variable* (or non-zero) *sum game* is a game in which the players have at least some common interests, so that a shift from one outcome to another can simultaneously help (or hurt) both players. A three-way classification of games with respect to the interests of the players is this: (i) *zero-conflict* coordination games, (ii) *zero-sum* or *total-conflict* games, and (iii) *variable-sum* games (with a mixture of common and conflicting interests), e.g., Battle of the Sexes, Prisoner's Dilemma, Chicken. *Note*: Contrary to what quite a few students said, zero-sum games need not be (and usually are not) "winner-take-all"/"loser take nothing." "Compromise" outcomes (or degrees of victory of defeat) may occur (think of playing poker or bridge for monetary stakes). [However, in a special kind of zero-sum game — called a *strong simple game* — one player (or coalition) must "win" and the other player (or complementary coalition) must "lose," with no intermediate possibilities.]

8. A zero-sum game is *strictly determined* if the maximin payoff for the maximizing player is equal to the minimax payoff for the minimizing player, i.e., the minimizing player can hold the maximizing player down to just what the latter can guarantee for himself (where both players are using pure strategies [see below]). In a strictly determined zero-sum game: (i) the maximin-minimax strategy pair is an *equilibrium*, (ii) strategic intelligence or deception is of no use against a rational opponent, and (iii) sequential play has no effect on the outcome. A zero-sum is *non-strictly determined*, if the maximin payoff to P1 is less than the minimax payoff to P2. This implies that there is no (pure strategy) equilibrium, there is an incentive for strategic intelligence and/or deception, and the second moving player has the advantage in sequential play. In such a game, both players should use *mixed strategies* (see below).

9. A *Prisoner's Dilemma* game is a 2×2 non-zero sum game in which both players have dominant strategies (necessarily in equilibrium) but both would do better if they both chose their dominated strategies. Their failure to realize this common interest cannot be corrected by mere pre-play communication (since an agreement to "cooperate," i.e., use their dominated strategies, is not an equilibrium); rather they must be able to make credible commitments to secure enforceable agreement.

3, 3	1, 4
4, 1	2, 2

10. Chicken is a 2 × 2 non-zero sum game in which neither player has a dominant strategy. They have a common interest in avoiding the mutual disaster that results when both "stand firm" but neither wants to be the one who "gives in." Thus each player wants to make the other believe that he is committed to "standing firm." The only equilibria are discriminatory, i.e., make one player the "winner" and the other the "loser."

0, 0	−2, +2
+2, −2	−10, −10

11. If players make *sequential moves* (with perfect information, i.e., the second moving player knows what strategy the first moving player has chosen), the first moving player should "*look ahead and reason back*" [D&N, Rule 1]. That is, the first-moving player should evaluate each of his strategies in terms of the *best reply* by the second-moving player and then choose the strategy that gives him the best payoff given a best reply by the other player. This "*backwards induction*" logic can be extended to perfect information games with any number of moves and/or players.

12. A *pure strategy* is a definite and complete plan of action for playing a game and, in a payoff matrix, corresponds to a row or column. A *mixed strategy* is a probability distribution over pure strategies. Both players in non-strictly determined zero-sum games have incentives to use mixed strategies. The maximizing player can increase his maximin payoff and the minimizing player can decrease his minimax payoff by using mixed strategies. Moreover, optimal mixed strategies are in equilibrium and restore the equality between the maximin and minimax payoffs characteristic of strictly determined games. An optimal mixed strategy always puts zero probability weight on dominated strategies. Iterated (repeated) non-strictly determined zero-sum games (e.g., D&N's discussion of pitcher-batter duels) make the desirability of mixed strategies especially evident.

13. In the standard formulation, players make their strategy choices simultaneously, not knowing each other's choice. *Pre-play communication* allows the players to communicate before making their strategy choices. Pre-play communication "solves" any pure coordination game by allowing the players to coordinate their strategy choices reliably. Furthermore, given their identical interests, neither player has any incentive to deceive the other. But if players have a (some) conflicting interests, they may have incentives to deceive one another or (try) to make credible commitments, threats, or promises.

14. In a bargaining situation (Generalized Chicken Game), one player may try to project an image that suggests he is crazy, irrational, emotional, and uncalculating and generally that he doesn't understand the risks of standing firm, thereby try to induce the other player to give in . This has been called the "*rationality of irrationality*"(or "the political uses of madness").

1. (**Total of 15 points**) Answer the following questions pertaining to the two two-player *zero-sum* games depicted in the payoff matrices below. (In each game, the row Player 1 has four strategies and the column Player 2 has three strategies. The number in each cell is the payoff to Player 1; the payoff to Player 2 is the negative of the number.)

Zero-Sum Game #1

	P2		
P1⇓	c_1	c_2	c_3
s_1	3	2	2
s_2	1	4	3
s_3	5	4	2
s_4	1	3	3

Zero-Sum Game #2

	P2		
P1⇓	c_1	c_2	c_3
s_1	5	2	3
s_2	4	5	3
s_3	3	4	1
s_4	4	3	2

(a) Draw a horizontal line through each of Player 1's *dominated* strategies (if any) in game #1.

s_1 dominated by s_3 and s_4 dominated by s_2

(b) Draw a vertical line through each of Player 2's *dominated* strategies (if any) in game #1.

c_2 dominated by c_3

(c) Draw a horizontal line through each of Player 1's *dominated* strategies (if any) in game #2.

s_3 and s_4 dominated by s_2

(d) Draw a vertical line through each of Player 2's *dominated* strategies (if any) in game #2.

c_1 dominated by c_3

(e) For each game, what is P1's *maximin* strategy and P2's *minimax* strategy?

P1's maximin strategy in Game #1 (CIRCLE ONE) **s_1** s_2 **s_3** s_4

However, s_1 is dominated

P2's minimax strategy in Game #1 (CIRCLE ONE) c_1 c_2 **c_3**

P1's maximin strategy in Game #2 (CIRCLE ONE) s_1 **s_2** s_3 s_4

P2's minimax strategy in Game #2 (CIRCLE ONE) c_1 c_2 **c_3**

(f) Is Game #1 strictly determined? Explain your answer.

No, because maximin for P1 = 2 < 3 = minimax for P2 (and there is no Nash Equilibrium)

(e) Is Game #2 strictly determined? Explain your answer.

Yes, because maximin for P1 = 3 = 3 = minimax for P2 (and there is a Nash Equilibrium)

2. (**3 points per question — total of 10 points**) Answer the following questions pertaining to the (variable-sum) game depicted in the payoff matrix below. (Each player has just two strategies. The number in lower-left corner of each cell is the payoff to the row Player 1; the number in the upper-right corner of each cell is the payoff to the column Player 2.)

		Player 2	
		c_1	c_2
Player 1	s_1	2 1	3 5
	s_2	4 3	5 3

(a) What do you expect the outcome of the game to be if the players must make their strategic choices *simultaneously* (not knowing what choice the other is making)? Explain briefly.
For P2, c_1 is dominated by c_2, so P1 expects P2 to choose his dominant strategy c_1. P1's best reply to c_1 is s_1. So we expect the outcome (s_1,c_1) giving payoffs (5,3). (Maximax gives the "right answer" but only by chance. Maximin is not good because the game is non-zero-sum.)

(b) What do you expect the outcome of the game to be if the players make their strategic choices *sequentially*, with *Player 1 moving first* and Player 2 second? Explain briefly.

P1 should look ahead and reason back. P1 sees that c_2 is P2's best reply to s_1, giving P1 a payoff of 5, and that c_2 is also P2's best reply to s_2, giving P1 a payoff of 3. So P1 chooses s_1 and P2 chooses c_2, giving payoffs (5,3).

(c) What do you expect the outcome of the game to be if the players make their strategic choices *sequentially*, with *Player 2 moving first* and Player 1 second? Explain briefly.

P2 should look ahead and reason back. P2 sees that s_2 is P1's best reply to c_1, giving P2 a payoff of 4, and that s_1 is P1's best reply to c_2, giving P2 a payoff of 3. So P2 chooses c_1 and P2 chooses s_2, giving payoffs (3,4). (Note: c_1 is not dominant in the sequential choice variant of the game because P1, as the second-moving player, has additional strategies available since his choice can be contingent on P2's choice.)