Problem Set #1: Apportionment

1. Overview

Apportionment formulas are used to apportion:

1. House seats among states according to their populations (*federal apportionment*); or
2. parliamentary seats among parties according to their vote support (*partisan apportionment* in *proportional representation* systems); or possibly
3. electoral votes within a state among Presidential candidates according to their vote support (as proposed in Colorado’s Proportion 36 in 2004).

Here we will use the language of federal apportionment, i.e., the apportioning House seats to states on the basis of their populations.

In class (and the PowerPoint presentation), two methods of apportionment were briefly discussed.

A *quota method* of apportionment is based on state *quotas* defined as follows:

(a) compute each state’s proportion of the total population; then
(b) multiply each state’s proportion of the population by the number of House seats to get each state’s quota.

The ideal of proportionality would be fully realized if every state could be awarded a number of electoral votes exactly equal to its quota. But, given that seats must be awarded in whole numbers while quotas (essentially) always entail fractions, exact proportionality is (essentially) never possible. Different apportionment methods prescribe different ways of “curing the mischief of fractions.”

The most common quota method of apportionment in the “Largest Remainder” method originally proposed by Alexander Hamilton for apportioning U.S. House seats among the states “according to their respective numbers.” (In European PR systems, it is commonly called the “Hare Largest Remainder” or “d’Hondt” method.) Hamilton’s method works as follows:

(a) each state is initially awarded its quota rounded down to the next integer; and
(b) the remaining seats (if any) are awarded to states in order of the size of the fractional “remainders” in their quotas to produce the final apportionment.

Colorado’s Proposition 36 proposed the following apportionment method:

(a) each state is initially awarded seats votes equal to its quota rounded up or down (in the normal manner) to the nearest integer;
(b) if the resulting allocation equals the actual number of seats to be awarded, this is the final apportionment; but...
(c) if “rounding error” means that too few seats have been initially awarded, the additional electoral votes are awarded to the largest state; and

(d) if “rounding error” means that too many electoral votes have been initially awarded, the excess electoral votes are taken as necessary from the state initially awarded the fewest electoral votes, then from the state initially awarded the second fewest electoral votes loses one to the leading candidate, and so forth.

Note. Colorado Proposition 36 was noted in class and discussed in one reading in the original packet, but the actual apportionment method was not specified. The *National Popular Vote* book discusses the Colorado proposal under the rubric of “the Whole-Number Proportional approach.” However, their assumes that there are only two Presidential candidates (or “states”), in which event all apportionment formulas are equivalent and simply round quotas off in the normal manner.

The other apportionment method discussed in class was the Jefferson (or Greatest Divisor) method, which is a “divisor” rather than “quota” method. All divisor methods award House seats sequentially to states on the basis of their “priority” for an additional seat. They differ in how this priority is defined. The Jefferson method works as follows:

(a) The first House seat is awarded to the largest state.

(b) The second House seat is also awarded to the largest state if its population divided by 2 is greater than the population of the second largest state; otherwise it is awarded to the second largest state.

(c) In general, each additional seat is awarded to the state with the “strongest claim” to the seat, where this claim is determined by the population of the state divided by the number of seats it has already been awarded plus one. [Other divisor methods divide by different numbers.]

The Jefferson Method of apportionment can also be carried out in the following (equivalent) manner.

(a) Divide the total population by the number of House seats to get the average Congressional District population. This is the inverse of the “ratio of representation” that the framers of the Constitution discussed and is called the *divisor*.

(b) Divide each state’s population by the divisor. The result is the state’s *quotient*.

(c) Round each quotient down to the next integer. [Other divisor methods use other rounding rules.] This gives an initial apportionment.

(d) If this does not allocate all seats (as will almost certainly be the case), reduce the size of the divisor until all House seats are apportioned.

*Note.* These descriptions (and the following problems) ignore the constitutional requirement that every state receive at least one seat.
2. \textit{Problems}

Here is the population of states in a small federal system

\begin{center}
\begin{tabular}{ll}
\text{State A} & 166,329 \\
\text{State B} & 109,859 \\
\text{State C} & 76,454 \\
\text{State D} & 46,723 \\
\text{State E} & 27,716 \\
\text{State F} & 23,920 \\
\end{tabular}
\end{center}

There are 15 House seats to be apportioned among the states (with no guarantee that every state will get at least one House seat).

Apportion the House seats according to each of the three methods discussed above. Then recalculate the Hamilton and Jefferson apportionments when the size of the House is increased to 16.

Here are some hints to help you started and to confirm you are on the right track.

(1) You should find that (with a House size of 15), state A has a quota of (approximately) 5.532 and State F has a quota of 0.796. These round down to 5 and 0 respectively.

(2) Under Jefferson’s Method, State A is awarded the first House seat; State B is awarded the second seat because its total population is greater than State A’s population divided by 2 (i.e., \( n + 1 \) where \( n \) is the number of seats A’s has already been awarded. State A is awarded the third seat, State C the fourth, and State A the fifth.

(3) If you use the alternate method for calculating the Jefferson apportionment, the initial divisor is 451,000/15 or about 30,067, but you will find you need to reduce the size of the divisor considerably to apportion all 15 seats.

\textit{Note.} You will certainly want to use a calculator to do this assignment. For Jefferson’s method, a spreadsheet program such as Excel (or SPSS) can be helpful.