RANDOM SAMPLING

Key Definitions Pertaining to Sampling

1. ***Population*****:** the set of "units" (in survey research, usually *individuals* or *households*), N in number, that are to be studied. A typical population in political science survey research, such as the National Election Studies, is the American *voting age population* (VAP).

2. ***Sample:*** any subset of units, n in number, drawn from the population. Almost always n is (much) smaller than N (but, perhaps surprisingly a sample can be larger than the population).

3. ***Sampling fraction*****:** the ratio n/N, i.e., the size of the sample in relation to the population. In most survey research, the sampling fraction is *very* small. (In national surveys such as ANES, it is on the order of 1/100,000.)

4. ***(Simple) Sampling Frame*****:** a list of every unit in the population.

5. ***Random (or Probability) Sample:*** a sample such that each unit in the population has a *calculable* (according to the laws of probability) chance of appearing in it — that is, a sample selected by a random mechanism (such as a lottery).

6. ***Non-Random Sample*****:** a sample selected in any non-random fashion, so that the probability that a unit is drawn into the sample cannot be calculated.

7. ***Simple Random Sample (SRS)*****:** a sample of size n such that every subset of n units has the same chance of constituting the sample. This implies that every individual unit has the same chance of appearing in the sample.

8. ***Systematic Random Sample*****:** a random sample of size n drawn from a simple sampling frame, such that each of the first N/n units on the list has the same chance of being selected and every $(N/n)^{\text{th}}$ subsequent unit on the list is also selected. This implies that every unit — but not every subset of n units — in the population has the same chance of being in the sample.

9. ***Multi-Stage Random Sample*****:** a sample selected by random mechanisms in several stages, most likely because it is impossible or impractical to acquire a list of all units in the population (i.e., because no simple sampling frame is available).

10. ***(Population) Parameter*****:** a characteristic of the population, e.g., the *percent of the population* that approves of the way that the President is handling his job, or the average household income in the population. For a given population at a given time, the value of a parameter is *fixed* but is typically *unknown* (which is why we may be interested in survey sampling).

11. ***(Sample) Statistic*****:** a characteristic of a sample, e.g., the *percent of a sample* that approves of the way that the President is handling his job, or the average household income in the sample. A sample statistic is typically used to *estimate* the comparable population parameter. The value of a sample statistic is *known* (for any particular sample) but it is not *fixed* — it

varies from sample to sample (even when the samples are all drawn from the same population with a fixed parameter value).

(a) Most population parameters and sample statistics we consider are *percentages*, e.g., the percent of the population or sample who approve of the way the President is doing his job, or the percent of the population or sample who intend to vote Republican in the upcoming election.

(b) A sample statistic is ***unbiased*** if its expected value is equal to the corresponding population parameter. This means that as we take repeated samples from the same population, the average of all the sample statistics "converges" on (comes closer and closer to) the population parameter.

(c) A sample statistic has more ***variability*** the more it varies from sample to sample.

12. ***(Random) Sampling Error:*** the magnitude of the inherent variability of sample statistics (from sample to sample). There are various ways of reporting sampling error. Public opinion polls and other surveys commonly report their sampling errors in terms of the *margin of error* associated with sample statistics. This measure of sampling error is defined and discussed below.

Important Points Pertaining to Sampling (with references to the attached Table of Sampling Results)

1. Sampling is indispensable for many types of research, in particular public opinion and voting behavior research, because it is impossible, prohibitively expensive, or self-defeating to study every unit in the population.

2. Many types of sampling (convenience, self-selected, haphazard, interviewer-selected, quota) are non-random and give no assurance of producing samples that are representative of the populations from which they are drawn. (Indeed, it often is not clear how to define the population from which such non-random samples are drawn.)

3. Random or probability sampling does provide an *expectation* of producing a representative sample, in the sense that random sampling statistics (or adjusted versions thereof) are *unbiased* (i.e., *on average* they equal true population parameters) and they are subject to a *calculable* (and *controllable*, by varying sample size and other factors) degree of *sampling error*, reflected in the fact that repeated random samples from the same population produce varying sample statistics. (See the enclosed Table of Sampling Results.)

4. More formally, most sample statistics are (approximately) *normally distributed* (we will introduce this concept formally in a few weeks) with an *average value* equal to the corresponding population parameter and a *variability* (sampling error) that (i) is mainly a function of *sample size n* (as well as variability within the population sampled) and (ii) can be *calculated* on the basis of the laws of probability.

The magnitude of sampling error can be expressed as the *standard deviation* (another concept we will introduce soon) or the *average absolute deviation* of sample statistics. (See the enclosed Table of Sampling Results.) More commonly, however, sampling error is expressed in terms of a *margin of error* of ± *X* %. The margin of error ± *X* % gives the magnitude of the *95% confidence interval* for the sample statistic, which can be interpreted in the following way.

Suppose the Gallup Poll takes a random sample of *n* respondents and reports that the President's current approval rating is 62% and that this sample statistic has a margin of error of ± 3 %. Here is what this means: if (hypothetically) Gallup were to take a great many random samples of the same size *n* from the same population (e.g., the American VAP on a given day), the different samples would give varying statistics (approval ratings), but *95% of these samples would give approval ratings within 3 percentage points of the true population parameter*, i.e., the Presidential approval rating we would get if we took a complete and wholly successfully *census* to get the opinion of every member of the American VAP. Put more practically (given that Gallup takes just one sample), we can be 95% confident that the actual sample statistic of 62% lies within 3 percentage points of the true parameter; i.e., we can be 95% confident that the President's "true" approval rating lies within the range of 59% (62% − 3%) to 65% (62% + 3%).

5. Considering the example above, you may well ask: how can the Gallup people say that its poll has a margin of error of ± 3% when they actually took just *one* poll, not the repeated polls hypothetically referred to above? The answer is that, given *random* samples, such margins of error can be *calculated* mathematically, using the laws of probability (in the same way one can calculate the probability of being dealt a particular hand in a card game or of getting particular outcomes in other games of chance). (See the attached page on Theoretical Probabilities of Different Sample Statistics.) This is the sense in which the *margin of error of random samples is calculable*, but that of a non-random sample is not.

6. Such mathematical analysis shows that random sampling error is (as you would expect) *inversely* (or *negatively*) related to the size of the sample — that is, smaller samples have larger sampling error, while larger samples have smaller error. However, this is not a *linear* relationship, e.g., doubling sample size does not cut sampling error in half. Rather sampling error is *inversely* related to the *square root* of sample size. Thus, if a given random sample has a margin of error of ± 6%, we can reduce this margin of error by increasing the sample size, but it will take a sample *four* times as large to cut the error in *half* (to ± 3%). In general, if Sample 1 and Sample 2 have sizes n_1 and n_2 respectively, and sampling errors e_1 and e_2 respectively, we have this relationship (the *inverse square root law*):

$$\frac{e_2}{e_1} = \frac{\sqrt{n_1}}{\sqrt{n_2}} \qquad (1)$$

Actually, for *simple* random samples and sample statistics that are percentages (e.g., percent approving of the way the President is doing his job), the following is approximately true:

(2) $$\textit{margin of error } (95\,\% \text{ confidence interval}) \approx \frac{100\,\%}{\sqrt{n}}.$$

See the entries in first column of Table 3.4 on p. 72 of Weisberg et al. (If parameters, statistics, and errors are given as decimal fractions, rather than percentages, this formula becomes: *margin of error* = $1/\sqrt{n}$.) Actual national surveys use random — but not simple random — samples, and their margins of error are slightly larger; see the remaining columns in Table 3.4.

Note. The values given in Table 3.4 on p. 72 of Weisberg et al. (and given by the approximate formula noted above) are the *maximum* sampling errors associated *non-extreme* parameter values. If the population parameter is fairly extreme, e.g., less than 10% or more than 90% (so that the population is quite homogeneous with respect to the variable of interest), sampling error actually somewhat less than that given in Table 3.4 or by the approximate formula. At the limit, if the population parameter is as extreme as possible, i.e., 0% or 100% (so that the population is perfectly homogeneous with respect to the variable of interest), the corresponding sample statistics necessarily have zero sampling error.

7. This inverse square root law has two important implications.

   a. *Increasing sample size* is subject to *diminishing marginal returns*. While one can always reduce sampling error further by increasing sample size, additional increments in n "purchase" less and less in terms of reducing sampling error. Quite small samples may have manageable sampling errors and additional research resources are usually better invested in reducing other types of (non-sampling) errors (see #11 below). For some purposes, a sample of about 1000 will achieve all the accuracy needed (e.g., a margin or error of ±3-4%). For many purposes, a sample of about 2000-3000 is sufficient.

   b. *Sample statistics for population subgroups have larger margins of error* than those for the whole population. For example, if a poll estimates the President's popularity in the public as a whole at 62 % with a margin of error of about ±3 %, the same poll estimates his popularity among men (or women) only with a margin of error of about ± 4.5 % (the relevant sample size is cut in half, so the margin of error is increased by a factor of $\sqrt{2}$ or about 1.5) and the estimate of his popularity among African-Americans only has a margin of error of about ±9 % (the relevant sample size is cut to about one-ninth, so the margin of error is increased by a factor of about 3). If one's research focuses importantly on such subgroups, it is desirable to use either (i) a larger than normal sample size or (ii) a *stratified sample* (see second to last point in #10 below).

8. There is a important a counterintuitive implication of this discussion, from the approximate formula (2) above, and from Table 3.4 in the Weisberg book. Notice that none of these makes any reference to the *population size* N (or to the sampling fraction n/N), as opposed to sample size n. This is because — for the most part — sampling error depends on *absolute sample size* (as well as variability within the population sampled), and *not* on sample size

relative to population size (i.e., the sampling fraction). This statement is precisely true if samples are drawn *with replacement*, i.e., if it is theoretically possible for a given unit in the population to be drawn into the same sample two or more times. Otherwise, i.e., if samples are drawn *without replacement* [which is the more common practice], the statement is true for all practical purposes, unless the sampling fraction is quite large, e.g., something like1/100 or larger. In survey research, of course, the sampling fraction is typically *much* smaller than this (for the NES, on the order of 1/100,000). Finally, if in fact we do draw a sample without replacement and with a high sampling fraction (e.g., 1/10), the only "problem" is that sampling error will be *less* than formula (2) and Table 3.4 indicate. (Of course, if the sampling fraction is 1 [i.e., $n = N$] and the sample is drawn without replacement, sampling error is zero (we have taken *census* of the population). On the other hand, note that, if we sample with replacement, sample size can increase without limit and, in particular, can exceed population size.)

An important implication of this fact is that, if a given margin of error is desired, a local survey requires essentially the same sample size as a national survey with the same margin or error. Thus, in so far as (interviewing, etc.) costs are proportionate to sample size, good local surveys cost almost as much as national ones.

9. A random sample may be selected by drawing cases from the sampling frame (list of units in the population) by some random or chance mechanism. Usually a list of random numbers is used. (See the attached Excerpt from a Table of Random Numbers.) However, you can go the the POLI 300 web page and can click on the link to *Research Randomizer* or to *Statistical Applets* and select Simple Random Sample. (The latter is recommended and described more fully in Problem Set #2.)

10. Because simple sampling frames (lists) do not exist for most large populations of individuals (particularly including the American VAP), simple random sampling often cannot be implemented. (A national SRS would also entail enormous personal interviewing costs, because the selected respondents would be scattered among thousands of locations.) *Multi-stage* (and consequently *clustered* and often *stratified* as well) samples are used instead.

    Suppose we want to study the attitudes of the population of American college students, by interviewing a representative sample of 2000 students. No simple sampling frame exists, i.e., there is no list of all 12,000,000 or so American college students. However, there are (pretty good) lists of all of the several thousand American colleges and universities, and these lists also show the (approximate) number of students enrolled in each. Using this list we can select a random sample of (say, 100) colleges and universities, where each institution has a *probability* of being drawn into this *first-stage* sample that is *proportional* to its *enrollment*. Then we would contact the Registrar's Office at each of the 100 selected institutions to get a list of the students enrolled in that school, and we would then use each list as a sampling frame to select a small simple random sample of 20 students from each of the 100 institutions. (It might turn out that some of the enrollment figures used to determine the probability of selecting institutions in the first stage of sampling are wrong. In this case, individual respondents might be *weighted* in the final sample to compensate for this error.) The final result is a *multi-stage* (in this case, a *two-stage*) random sample of 2000 American college students. The sample is also *clustered* in that the 2000 student respondents are clustered on just 100 campuses, rather than spread out over almost 2000 different campuses (as would be true if we had a SRS of 2000 students). Clustering has the advantage of greatly reducing personal interviewing costs. Statistics from such a clustered multi-stage sample are unbiased, though they have somewhat greater sampling error than those from a simple random sample of the same size, which can be compensated for by increasing sample size somewhat. (Note that we could have selected a *simple random sample* of colleges, i.e., by not weighting probabilities of selection by enrollments, and then used the same *sampling fraction* at each selected college. This would also produce an unbiased two-stage random sample; however, its sampling error would be considerably greater than that resulting from the procedure recommended above.)

We might also *stratify* the sample by selecting separate samples of appropriate size (totaling 2000) from (for example) (a) community colleges, (b) four-year colleges, and (c) universities, and/or from different regions of the country, etc. Such stratification, where feasible, reduces sampling error compared with non-stratified samples of the same size. Stratification is especially useful if we want systematically to *compare two subgroups of unequal size* (e.g., whites and blacks, partisans and independents). In this event, it is desirable to stratify by subgroups and draw samples of (approximately) equal size for each subgroup (so that the sampling fraction is inversely related to group size), with the result that statistics for each subgroup are subject to (approximately) the same margin of error.

See Weisberg et al., pp. 49-61, for a more detailed discussion of sampling methods used to conduct such large-scale national surveys of the VAP as the ANES. (In contrast, the British Election Studies use the national list of enrolled voters as a simple sampling frame for a one-stage non-clustered national sample stratified by region (Scotland, Wales, etc.).)

11. Survey research is subject to many types of error in addition to sampling error.

Such *non-sampling errors* include most importantly errors resulting from a low *response* (or *completion*) *rate*. Not every person drawn into the sample by random chance can be successfully interviewed. Some people in the drawn sample may never be located, may never be at home, or may simply refuse to submit to the interview. A low completion rate reduces the size of the *completed sample*, and thus increases sampling error. *Much more importantly*, non-respondents, in considerable measure, are *self-selected* or otherwise *not randomly selected* out of the drawn sample. Thus the completed sample is not a random sample of the drawn sample nor a fully random sample of the population as a whole, and its sample statistics may be *biased* in more or less unknown ways.

Other non-sampling errors include:

a. *non-coverage error* (the sampling frame may not cover exactly the population of interest);

b. *measurement errors* due to unambiguous, unclear, or otherwise poorly framed questions or poorly designed questionnaires, inappropriate interviewing circumstances, interviewer mistakes, etc.; and

c. data entry, coding, tabulation, or other data processing errors.

Note that all these are indeed *non-sampling errors* — data based on a complete census of the population would be subject to the same errors, which therefore cannot be blamed on the sampling process. Once sample size reaches a reasonable size (which may depend on the type of research being done), extra resources are better devoted to increasing the response rate and reducing other kinds of non-sampling errors than to further increasing sample size.

Using SPSS to Draw Random Samples from the SETUPS Data with a Known Population Parameter

Note. I have not updated this exercise using SETUPS 1972-2004 data.

The SETUPS 1972-2000 data pools together samples from each of the eight National Election Studies in the period covered. Each NES study has a sample size of approximately $n \approx 2000$. Pooled together, there are 16,438 respondents in the entire study. Let us consider this set of units people to constitute a *population* with $N = 16{,}438$ (a population size comparable to the VAP of a small city). The SPSS (Statistical Package for the Social Sciences) computer program (to which you have been introduced) has a procedure that allows the researcher to draw simple random samples of any size from the entire data set available for analysis. (See the end of Section VII of the SPSS handout.)

First, let us use SPSS to calculate the value of a particular *population parameter* — say, the percent of respondents in the population who give the "approve" answer to the question "Do you approve or disapprove of the way the President is handling his job?" as a percent of the number of people who answered the question. This is variable V29 (PRESIDENTIAL JOB APPROVAL) in the SETUPS Codebook. We determine the population parameter by calculating the following, on the basis of all 16,438 responses:

$$\textit{parameter} = \frac{\textit{number of respondents coded } \text{“1”}}{\textit{number coded } \text{“1”} + \textit{number coded } \text{“2”}} \times 100\% = 58.5\% .$$

(This excludes 2385 cases coded "9" as "missing data.")

Normally, of course, we don't know the value of a population parameter, which is precisely why we resort to survey research using sampling. In this exercise, however, we do know the value of the parameter, and we sample anyway, so that we can actually check how accurate the sampling is.

I took 20 samples of size $n = 15$, 20 samples of size $n = 150$, and 20 samples of size $n = 1500$. (I took "samples of samples," if you will.) By the inverse square law, sampling error should be greatest in the smallest samples and smallest in the largest samples. According to the approximate formula (2) given in #6 above, the margin of error in samples of size $n = 1500$ is about $\pm 2.6\%$. (The Table 3.4 in Weisberg et al. says the same.) This sample size and corresponding margin of error are typical of much survey research. Likewise, the margin of error in samples of size $n = 150$ is about $\pm 8.2\%$. (The Weisberg table gives an interpolated value of about $\pm 9\%$.) This sample size is typical of a number of *subgroups* in a VAP sample of about 1500, e.g., African-Americans, Hispanics, non-Christians, (pure) Independents, etc., each of which constitutes about 10% of the total population. The margin of error in samples of size $n = 15$ is about $\pm 25.8\%$. Such samples are extremely small and their statistics obviously have very high margins of error, and few social scientists would venture to make inferences from them.

The resulting sample statistics are shown in the Table of Sampling Results at the end of this handout. The table shows at total 60 sample statistics arranged in three columns, 20 for $n = 15$, 20 for $n = 150$, and 20 for $n = 1500$. Each of the 60 samples was independently selected, so the order in which they are listed (and numbered) is arbitrary and there is no connection between (for example) the 10th sample of size 15 and the 10th sample of size 150 (or 1500). The column to the right of each sample statistic shows the amount by which the sample statistic *deviates* (differs) from the true population parameter of 58.5%; the deviation is positive if the statistic is greater than the parameter and negative if the statistic is smaller than the parameter.

These data clearly illustrate the two theoretical points about random sampling set out above — namely that (i) such sample statistics are *unbiased* but also that (ii) they are subject to *sampling error* that is *inversely related to the square root of sample size*.

With respect to the first point, we see from the data that, regardless of sample size, the sample statistics are just about right *on average* (60.2%, 57.0%, 58.5%); equivalently the deviations add up to just about zero. This reflects the fact that the sample statistics (regardless of sample size) are *unbiased*. Had we taken a larger (than 20) "sample of samples" of each size, average performance of the sample statistics (especially those from the smallest samples) would be even better. On the other hand, the fact the 20 statistics from the largest samples appear to be *exactly* right on average is a merely coincidence (and in any event is an illusion resulting from rounding to the nearest one tenth of a percent).

At the same time, we also see that (almost) every individual sample statistic *deviates* at least a bit from the true population parameter (and even the ones that appear to be right on the mark of 58.5% are really off a bit — the discrepancy doesn't show up because of rounding), about half being too low (negative deviations) and half too high (positive deviations), reflecting the fact that sample statistics are subject to *sampling error*. Moreover, it can be seen that this sampling error is inversely related to sample size and very closely follows the *inverse square root law*. The sample sizes are in a ratio of 1 to 10 to 100, so by the inverse square root law the associated sampling errors should be in a ratio of 10 to $\sqrt{10}$ to 1. The ratios of either the *mean absolute* (i.e., ignoring "+" and "−" signs) *deviations* or the *standard deviations* associated with each sample size closely duplicate these ratios.

The probability calculations described above tell us that, among the largest samples ($n = 1500$), the margin of error is about $\pm 2.6\%$. Remember that this means we expect that on average about 19 sample statistics out of the 20 (95 %) will fall within ± 2.6% of the population parameter, i.e., within the interval 58.5% ± 2.6% (or 55.9–61.1%). In fact, all our sample statistics fall within this interval, though one (#1) falls close to the upper bound of the interval. Likewise, the calculations lead us to expect that, among the medium-sized samples (n = 150), the margin of error is about ± 8.2 %. That is, we expect that about 19 sample statistics out of the 20 will fall within ± 8.2% of the population parameter, i.e., within the interval 56.3% ± 8.2% (or 50.3–66.7%). In fact, all but three sample statistics (#9, #13, #16) fall within this interval. Finally, the calculations lead us to expect that, among the smallest samples (n = 15), the margin of error is about ± 25.8%. That is, we expect that about 19 sample statistics out of the 20 will fall within ± 25.8% of the population parameter, i.e., within the interval 56.3% ± 25.8% (or 32.7–84.3%). In fact, all but one sample statistic (#5) fall within this interval. All together, we expect 95% of the 60 sample statistics (all but three) to fall within their respective margins of error. In fact, all but four statistics do so.

This sampling data is also presented in graphical form below. Each sample statistic is plotted as a little box (■) on the horizontal line corresponding to its sample size. (The boxes merge into each other where sample statistics are almost equal.) The true population parameter is shown by the vertical line at 58.5 on the horizontal axis. It is immediately evident that the boxes on the top (n = 1500) line are closely concentrated around the population parameter. The boxes along the middle (n = 150) line are considerably more spread out and those on the bottom (n = 15) line are still more spread out. However, on each line, the boxes on either side of population parameter approximately balance out.

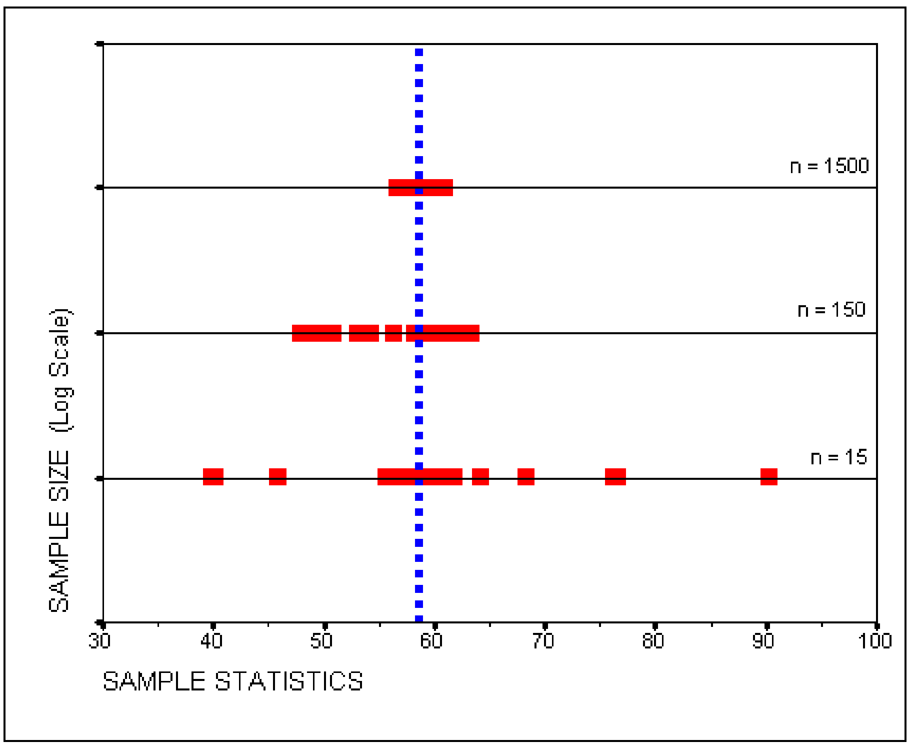

TABLE OF SAMPLING RESULTS

Population parameter = **58.5% (V29 Presidential Approval)**

Table shows samples statistics for 20 samples of each size

Sample #	n = 15	(Dev.)	n = 150	(Dev.)	n = 1500	(Dev.)
1	56.3	−2.2	61.0	+2.5	60.9	+2.4
2	58.1	−0.4	61.9	+3.4	57.3	−1.2
3	61.8	+3.3	61.2	+2.7	59.0	+0.5
4	61.4	+2.9	63.3	+4.8	57.5	−1.0
5	90.2	+31.7	59.9	+1.4	58.7	+0.2
6	39.8	−18.7	60.3	+1.8	60.5	+2.0
7	60.2	+1.7	58.5	0.0	59.1	+0.6
8	64.1	+5.6	54.2	−4.3	57.5	−1.0
9	56.0	−2.5	49.4	−9.1	59.9	+1.4
10	76.5	+18.0	60.1	+1.6	58.8	+0.3
11	40.2	−18.3	61.5	+3.0	58.2	−0.3
12	57.8	−0.7	53.4	−5.1	58.8	+0.3
13	76.2	+17.7	47.9	−10.6	58.2	−0.3
14	59.8	+1.3	58.2	−0.3	57.5	−1.0
15	61.4	+2.9	60.5	+2.0	58.5	0.0
16	56.5	−2.0	49.6	−8.9	58.0	−0.5
17	68.2	+9.7	53.0	−5.5	58.7	+0.2
18	55.5	−3.0	50.8	−7.7	56.6	−1.9
19	58.4	−0.1	56.3	−2.2	57.0	−1.5
20	45.7	−12.8	58.8	+0.3	59.5	+1.0
Mean	60.2	+1.7	57.0	−1.5	58.5	0.0
Mean Ab.Dev.	7.8	7.8	3.9	3.9	0.9	0.9
Standard Dev.	11.7	11.7	4.8	4.8	1.1	1.1

THEORETICAL PROBABILITIES OF DIFFERENT SAMPLE STATISTICS

Consider the following **population**: a deck of cards with $N = 52$. In this case, of course, we know all the characteristics (**parameters**) of this population (e.g., the percent of cards in the deck that are red, clubs, aces, etc.) we can consider what we **expect** will happen if we take repeated **random samples** (with replacement) of size $n = 2$ out of this population.

Example #1. Let the **population parameter** of interest be **the percent of cards in the deck that are red**. Suppose we try to estimate the value of this parameter using the corresponding **sample statistic**, i.e., **the percent of cards in the sample that are red**. While we know that the sample statistic will vary from sample to sample, we can calculate how likely we are to get any specific sample statistic using the laws of probability .

On any draw (following **replacement** on the second and any subsequent draws), the probability of getting a red card is .5 (since half the cards in the population are red) and the probability of getting a non-red (black) card is also .5 .

1^{st} draw	2^{nd} draw	Probability	Sample Statistic	Probability
R	R	.5 × .5 = .25	100%	.25
R	B	.5 × .5 = .25	50%	.50
B	R	.5 × .5 = .25		
B	B	.5 × .5 = .25	0%	.25

Example #2. Let the **population parameter** of interest be **the percent of cards in the deck that are diamonds**.

On any draw (following replacement on the second or subsequent draws), the probability of getting a diamonds card is .25 (since a quarter of the cards in the population are diamonds) and the probability of getting a non-diamond (hearts, clubs, or spades) card is .75 .

1^{st} draw	2^{nd} draw	Probability	Sample Statistic	Probability
◊	◊	.25 × .25 = .0625	100%	.0625
◊	0	.25 × .75 = .1875	50%	.3750
0	◊	.75 × .25 = .1875		
0	0	.75 × .75 = .5625	0%	.5625

Note that sampling **with replacement** greatly simplifies these calculations. If we sampled **without replacement**, given that (for example) we get a red card on the first draw, the probability of getting red card on the second draw is not .5 but 25/51 ≈ .49 and the probability of getting black card on the second draw is not .5 but 26/51 ≈ .51 .

Excerpt from a Table of Random Numbers

Line	(1)	(2)	(3)	(4)	(5)	(6)	(7)
251	89429	26726	15563	94972	78739	04419	60523
252	43427	25412	25587	21276	44426	17369	29010
253	58575	81958	51846	02676	67781	95137	88430
254	61888	71246	24246	23487	78639	92006	63846
255	73891	47025	40937	71907	26827	98865	38882
256	40938	73894	40854	15997	55293	95033	31736
257	98053	43567	17292	86908	71364	06089	92394
258	59774	29138	46993	39836	99596	59050	25419
259	09765	07548	63043	59782	81449	13652	94420
260	38991	64502	24770	29209	82909	66610	84418
261	25622	27100	56128	62145	82388	45197	97609
262	31864	74120	66231	82306	91784	33177	17681
263	81171	75639	60863	49562	28845	81581	10249
264	69874	52803	28544	51569	56090	44558	42095
265	27848	51107	05761	02159	53911	01952	59273
266	69407	69736	75375	31488	67528	84234	76462
267	29418	03091	06364	13151	40663	43633	87954
268	38222	31231	79415	44558	62490	26936	49682
269	94720	83796	93251	03568	62484	29140	14152
270	45275	16852	02284	41361	73733	61486	33189
271	97260	09552	82626	42915	45847	87401	13339
272	01990	65259	60684	78175	43825	45211	86287
273	24633	42314	81192	50253	67516	59076	92006
274	98071	52677	74920	74461	52266	26967	68284
275	34101	79442	28403	48541	13010	16596	72001
276	77186	93967	25910	66403	73837	73445	86663
277	23114	05481	42335	51396	60823	22680	50459
278	59988	49944	41038	99977	16348	41119	51548
279	11852	42254	82304	05588	75165	20179	94198
280	59992	87922	56299	01700	07003	97507	69260
281	42116	86593	22828	41422	18176	03250	06079
282	39663	61401	21471	42702	70588	53144	27087
283	53542	72009	96296	68908	58657	87117	21483
284	25996	76108	98476	36397	89457	19577	65877
285	91106	26450	14451	50328	29084	32332	08635
286	37133	88924	27845	13024	90687	23726	11212
287	13982	25736	10087	16762	02564	27250	79316
288	26663	36187	01688	25005	46677	75851	73938
289	62572	08275	16313	24936	81680	53829	40412
290	65925	95455	08383	24643	72962	08172	37824
291	97978	74676	08942	48919	51592	71196	48534
292	01914	42524	67820	47985	91773	10383	89514
293	68565	44811	39238	70394	78555	33539	56310
294	54370	31672	03893	32423	54092	69375	63308
295	79954	89601	23881	46951	69084	33477	87968
296	55479	01059	44229	56975	06785	80930	26443
297	38114	70330	42157	86699	46212	74692	92603
298	29766	83452	66202	02488	72704	97821	70614
299	31771	70640	34779	41831	33456	53194	19602
300	77528	87188	83577	99067	83835	48662	31503