HW 3 : MATH 301/02, Spring 2016, due in class on March 3rd Thu

The starred problems must be turned in. Please come prepared to discuss ALL the problems in class, especially the unstarred problems, as these are intended for practice!

Q1 If $a, b \in \mathbb{R}$, show that $|a + b| = |a| + |b|$ if and only if $ab \geq 0$.

Q2 If $a, b, c \in \mathbb{R}$ and $a \leq c$, show that $a \leq b \leq c$ if and only if $|a - b| + |b - c| = |a - c|$. Interpret this geometrically.

Q3 * Let $S \subseteq \mathbb{R}$ be bounded and have supremum $u$. Define the set $S_2$ by

$$S_2 = \{-x \mid x \in S\}.$$

Prove that $-u$ is the infimum of $S_2$.

Q4 * Let $A, B$ both be nonempty subsets of the reals and that $A \subseteq B$. Suppose that $b = \sup(B)$. Prove that $\sup(A)$ exists and that $\sup(A) \leq b$.

Q5 * Let $A, B$ both be nonempty subsets of the reals that are bounded above and that $a = \sup(A)$ and $b = \sup(B)$. Prove that $A \cup B$ is bounded above and that $\sup(A \cup B) \leq \max\{a, b\}$.

Q6 * Suppose $S \subseteq \mathbb{R}$ and let $u \in \mathbb{R}$. Show that $u = \sup(S)$ if and only if for each $n \in \mathbb{N}$, $u - 1/n$ is not an upper bound of $S$ and $u + 1/n$ is an upper bound of $S$. 