PHYS721–Atmospheric Radiative Transfer
Problem Set 1

1 – Given the angular diameter of the sun, as seen from earth, is 32 minutes of arc, calculate the total solid angle $\Delta \omega$ occupied by the sun in the sky.

2- Consider a horizontal snowfield on earth’s surface, with a flux reflectance (albedo) $r_{\text{snow}}$, illuminated by the sun at a zenith angle of 60°. Assuming that the snow reflects radiation isotropically, show that the brightness (equivalent to “intensity”, or “radiance” in units of $[\text{Wm}^{-2} \text{sr}^{-1}]$) of the snow, $I_{\text{snow}}$, relative to that of the sun $I_{\text{sun}}$, is:

$$\frac{I_{\text{snow}}}{I_{\text{sun}}} = r_{\text{snow}} \frac{\Delta \omega}{2\pi}$$

3 – A Lambertian reflector is a surface that provides equal reflectances in all directions (isotropic). Prove that for a Lambertian reflector surface, the reflected flux and radiance are related through $F^r = \pi I$.

4 – Consider a full moon (not eclipsed). Assuming that the moon has uniform albedo $r_{\text{moon}}$, and that all parts of the lunar surface reflects isotropically, the reflected brightness will vary across the illuminated hemisphere of the moon because the moon is a curved surface, so that different zones of the moon are illuminated at different zenith angles. Assume that the moon-sun distance is negligibly different from the earth-sun distance, and that the lunar radius is much smaller than the earth-moon distance. Show that the average brightness of the full moon, $I_{\text{moon}}$ (averaged over the lunar disk, as seen from the earth), relative to that of the sun, is:

$$\frac{I_{\text{moon}}}{I_{\text{sun}}} = \frac{4}{3} \left( r_{\text{moon}} \frac{\Delta \omega}{2\pi} \right)$$

5 - Evaluate the results of problems (2) and (4) to obtain the relative brightness of sun, snow, and moon, assuming that the snow and moon have albedos 1.0 and 0.1 respectively. Compare your value of $I_{\text{moon}}/I_{\text{sun}}$ to the measured value $1/430,000$ and give a possible reason for the difference.