# Solutions Homework 5 - Phys 224



5.2

$g = 9.8$ m/s

$\ell = .4$ m

(a) $\omega_0 = \sqrt{\frac{g}{\ell}} = 4.950$ rad/s

$T = \frac{2\pi}{\omega_0} = 1.269$ s

with one pendulum clamped:

$$\frac{d^2x}{dt^2} = -\omega_0^2 x - \frac{k}{m}x$$

call $(\omega')^2 = (\omega_0^2 + \frac{k}{m})$

However $T = 1.25$s $\quad \omega' = \frac{2\pi}{1.25s}$

Solving for $\frac{k}{m} = .7495$ Hz

The frequency we want is

$$\omega'' = (\omega_0^2 + 2\frac{k}{m})^{1/2} = 5.100 \text{ rad/s}$$

$$T'' = \frac{2\pi}{\omega''} = 1.231 s$$

(b) Using 5-7

$$x_A = A_0 \cos\left(\frac{\omega'' - \omega_0}{2} t\right)\cos\left(\frac{\omega'' + \omega_0}{2} t\right)$$

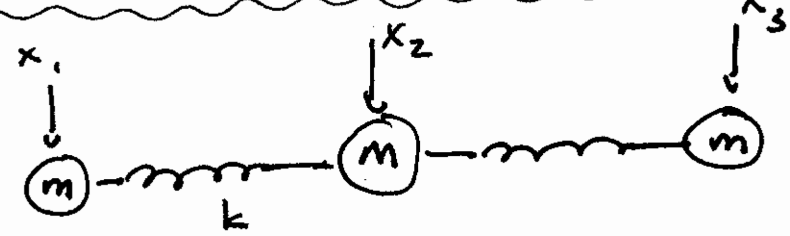
The low frequency envelope is given by $\cos\left(\frac{\omega'' - \omega_0}{2} t\right)$

Maximum amplitudes occur when

$\omega T = \pi$

Here $T = \dfrac{2\pi}{\omega'' - \omega_0} \approx 41.6$ s

5.9




Equations of motion

$$m \frac{d^2 x_1}{dt^2} = -k(x_1 - x_2)$$

$$m \frac{d^2 x_2}{dt^2} = -k(x_2 - x_3) - k(x_2 - x_1)$$

$$m \frac{d^2 x_3}{dt^2} = -k(x_3 - x_2)$$

assume

$$x_1 = a_1 \cos(\omega t) \qquad \frac{d^2 x_1}{dt^2} = -\omega^2 a_1 \cos(\omega t)$$

$$x_2 = a_2 \cos(\omega t) \qquad \frac{d^2 x_2}{dt^2} = -\omega^2 a_2 \cos(\omega t)$$

$$x_3 = a_3 \cos(\omega t) \qquad \frac{d^2 x_3}{dt^2} = -\omega^2 a_3 \cos(\omega t)$$

and write the matrix equation:

$$\begin{pmatrix} (k - m_1\omega^2) & -k & 0 \\ -k & (2k - m_2\omega^2) & -k \\ 0 & -k & (k - m_3\omega^2) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

or $A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$

consider $m_1 = m_3 = m$ and $m_2 = M$

These equations have a non-zero solution if $\det(A) = 0$

$$\det(A) = \begin{vmatrix} k - m\omega^2 & -k & 0 \\ -k & 2k - M\omega^2 & -k \\ 0 & -k & k - m\omega^2 \end{vmatrix} = 0$$

Note: $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$\det(A) = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$

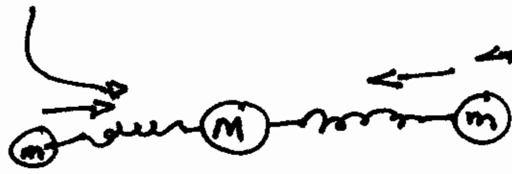
$$(k - m\omega^2)\begin{vmatrix} 2k - M\omega^2 & -k \\ -k & k - m\omega^2 \end{vmatrix} + k\begin{vmatrix} -k & -k \\ 0 & k - m\omega^2 \end{vmatrix}$$

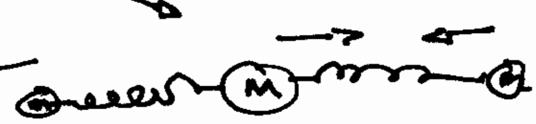
$$= (k - m\omega^2)\left((2k - M\omega^2)(k - m\omega^2) + k^2\right) - k^2(k - m\omega^2)$$

$$= \omega^2(m\omega^2 - k)(kM + 2km - Mm\omega^2) = 0$$

$$\omega = 0, \quad \omega = \sqrt{\frac{k}{m}}, \quad \omega = \left(\frac{k}{m} + 2\frac{k}{M}\right)^{1/2}$$

pure translation

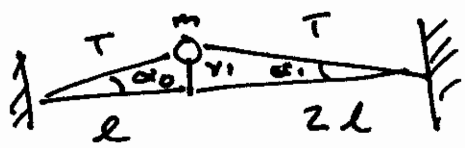




(b) $m = 16 \quad M = 12$

$$\frac{\left(\frac{k}{m} + 2\frac{k}{M}\right)^{1/2}}{\left(\frac{k}{m}\right)^{1/2}} = \left(1 + 2\frac{m}{M}\right)^{1/2} = \left(\frac{44}{12}\right)^{1/2} = \left(\frac{11}{3}\right)^{1/2} \approx 1.91$$

## 5-13

(a)




From the section on the coupled oscillator

$$\sin \alpha_0 = \frac{y_1}{l}$$

$$\sin \alpha_1 = \frac{-y_1}{2l}$$

$$m\frac{d^2 y_1}{dt^2} = -2T\frac{y_1}{2l} - T\frac{y_1}{2l} = -3T\frac{y_1}{2l}$$

$$\omega_0^2 = \frac{3T}{2ml} \qquad T = \frac{2\pi}{\omega_0} = 2\pi\left(\frac{2ml}{3T}\right)^{1/2}$$

(b) From Figure 5-11


lower

upper

(c) For the lower normal mode

$$\omega_0^2 = \frac{T}{ml} \qquad \text{From Fig. 5-11} \quad \omega_{upper} = \sqrt{3}\,\omega_0 = \left(\frac{3T}{ml}\right)^{1/2}$$

5-16

$$Y_{N+1} = h\cos(\omega t)$$

Try $A_p = C\sin(\alpha p)$

Determine $C$ and $\alpha$

Combine Equations 5-19 and 5-21

$$\frac{A_{N-1} + A_{N+1}}{A_N} = 2\cos\alpha = \frac{2\omega_0^2 - \omega^2}{\omega_0^2}$$

$$\cos(\alpha) = \left(1 - \frac{\omega^2}{2\omega_0^2}\right)$$ ← condition $\omega < 2\omega_0$ ensure this value is between zero and one

$$\alpha = \cos^{-1}\left(1 - \frac{\omega^2}{2\omega_0^2}\right)$$

$$Y_{N+1} = A_{N+1}\cos(\omega t) = h\cos(\omega t)$$

$$C\sin((N+1)\alpha)\cos(\omega t) = h\cos(\omega t)$$

$$C = \frac{h}{\sin((N+1)\alpha)}$$