# Chapter 4 Solutions

4.3 $m = .2\,kg$ $\quad k = 80 \frac{N}{m}$ $\quad \omega_1 = \sqrt{\frac{k}{m}} = 20\,Hz$

$b = 4 \frac{N}{m} s$

(a) $m \frac{d^2x}{dt^2} - b \frac{dx}{dt} + kx = 0$

$$\frac{d^2x}{dt^2} - \frac{b}{m} + \frac{k}{m} x = 0$$

using (3-34) $\omega_0 = \sqrt{\frac{k}{m} - \frac{b}{4m}} = 10\sqrt{3} \approx 17.3$ Hz

(b) $F_0 = 2N$ $\quad \omega = 30\,Hz$

use ~~eq~~ (4-11) $A(\omega) = 1.28\,cm$

4.5 (a) $\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \frac{g}{l} x =$ Driving Force $= \frac{g}{l} \xi$

↑ restoring force of simple harmonic motion

Let $z = A e^{j(\omega t - \delta)}$

$$\frac{dz}{dt} = A\omega e^{j(\omega t - \delta)}$$

$$\frac{d^2z}{dt^2} = -A\omega^2 e^{j(\omega t - \delta)}$$

$$(-\omega^2 A + \gamma A \omega j + \omega_0^2 A) e^{j(\omega t - \delta)} = \omega_0^2 \xi_0 e^{j\omega t}$$

equate real and imaginary parts

$$(\omega_0^2 - \omega^2)A = \omega_0^2 \xi_0 \cos\delta \quad \Rightarrow \quad A = \frac{\omega_0^2 \xi_0}{[(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2]^{1/2}}$$

$$\gamma \omega A = \omega_0^2 \xi_0 \sin\delta \qquad \tan(\delta) = \frac{\gamma\omega}{\omega_0^2 - \omega^2}$$

# Chapter 4 solutions

4.5

$x = A\cos(\omega t - \delta)$ with $A$ and $\delta$

as defined.

ⓑ $Q = \frac{\omega_0}{\gamma}$  (3-35) $A(t) = A_0 e^{-\frac{\gamma t}{2}} = A e^{-\frac{\omega_0 t}{2Q}}$

$T = \frac{2\pi}{\omega_0} \approx 2.00\,s$  $t \approx 100.3\ s$

$-\frac{\omega_0 t}{2Q} = -1 \longrightarrow Q = 50\pi = 157.08$

(413) $A(\omega_0) = \dfrac{F_0/m}{\frac{\omega_0^2}{Q}}$  $\frac{F_0}{m} = \omega_0^2 \xi$

$A(\omega_0) = \xi_0 Q = \frac{1}{20}\pi\ m \approx .157\ m$

ⓒ $\frac{1}{2}\frac{Q}{\omega_0^2} = \dfrac{1}{\left[(\omega_0^2 - \omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2}\right]^{1/2}}$

Just Solve For $\omega$. There is a lot of algebra.

$\omega = \pm 3.149, \pm 3.132$ Hz

$l = 1m$  $\sqrt{\frac{g}{l}} = 3.132$ Hz

Ignore the negative values and you have

$\omega = 3.149$ Hz or $3.132$ Hz

# Chapter 4 Solutions

4.6

(a) $$\frac{d^2y}{dt^2} + \gamma \frac{dy}{dt} + \omega_0^2 \frac{dy}{dt} = - \frac{d^2\eta}{dt^2}$$

no restoring force

(b) $\eta = C \cos(\omega t)$

$$A e^{j(\omega t - \delta)}(-\omega^2 + j\omega\gamma + \omega_0^2) = \omega^2 C e^{j\omega t}$$

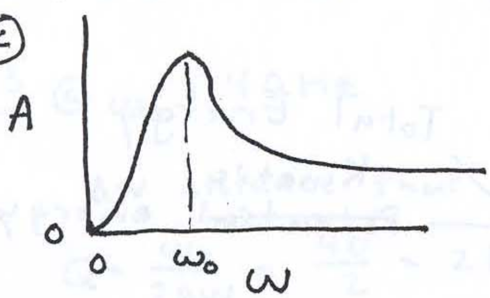
Procede as before

$$A(\omega_0^2 - \omega^2) = \omega^2 C \cos\delta$$

$$A\omega\gamma = \omega^2 C \sin\delta$$

$$A = \frac{\omega^2 C}{[(\omega_0^2 - \omega^2)^2 + \omega^2\gamma^2]^{1/2}} \qquad \tan\delta = \frac{\omega\gamma}{\omega_0^2 - \omega^2}$$

(c)




(d) $T_1 = 30\,s$ $\quad Q = 2$ $\quad T_2 = 1200$ $\quad \omega = \frac{2\pi}{1200}$

$$\gamma = \frac{\omega_0}{Q} = \frac{2\pi}{60} \qquad \frac{F_0}{m} = 10^{-9}\ \frac{m}{s^2}$$

use (4-11)

$$A(\omega) = 2.2804\cancel{4} \times 10^{-8} \approx 228.044\ \text{Å}$$

# Chapter 4 Solutions

ⓐ $W = \int F \cdot dx$ $\qquad x = A\sin(\omega t)$

$F = -b\frac{dx}{dt}$ $\qquad \frac{dx}{dt} = A\cos(\omega t)$

$dx = A\omega\cos(\omega t)\,dt$

$W = \int_0^{\frac{2\pi}{\omega}} -bA^2\omega^2\cos^2(\omega t)\,dt$ $\qquad u = \omega t$

$\frac{du}{\omega} = dt$

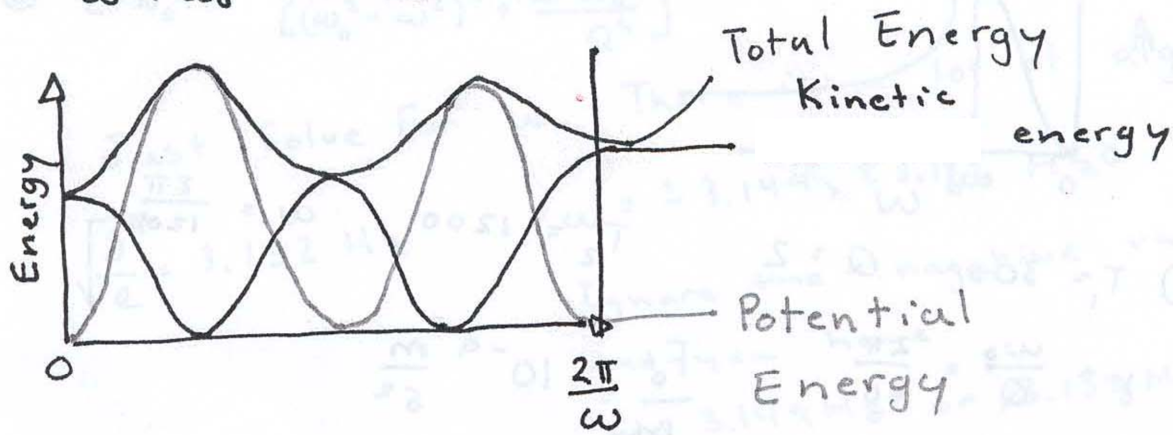
$= -bA^2\omega\int_0^{2\pi}\cos^2(\cancel{\omega t}\,u)\,du$

$= -bA^2\omega\pi$

ⓑ $K = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 = \frac{1}{2}m\omega^2A^2\cos(\omega t)$

$U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega_0^2A^2\sin^2(\omega t)$

$\omega < \omega_0 \rightarrow \omega^2 < \omega_0^2$



# Chapter 4 Solutions



4-12

(a) $m = 2kg \qquad Q = 15 \qquad x = .025m$

$\frac{mg}{x} = k = 784.532 \frac{N}{m}$

$\omega_0 = \sqrt{\frac{k}{m}} = 19.8057 Hz$

(b) $F_0 = C\omega_0^2$ using $C = 1mm$

(4-11) $A(\omega_0) = \frac{\omega_0^2 C}{\frac{\omega_0^2}{Q}} = CQ = .015m$

(c) using (4-26)

$\bar{P}(\omega) = \frac{\gamma F_0^2}{2m} \frac{1}{4(\omega_0-\omega)^2 + \gamma^2} \qquad \gamma = \frac{\omega_0}{Q}$

$\bar{P}(1.02\omega_0) = \frac{\gamma F_0^2}{2m} \frac{1}{4(.02\omega_0)^2 + \gamma^2} \approx .0856 \quad W$